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Solving Quadratic Equations by Completing the Square



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Perfect Square Trinomials

- Examples

- $x^2 + 6x + 9$

- $x^2 - 10x + 25$

- $x^2 + 12x + 36$

Creating a Perfect Square Trinomial

- In the following perfect square trinomial, the constant term is missing.
 $x^2 + 14x + \underline{\hspace{2cm}}$
- Find the constant term by squaring half the coefficient of the linear term.
 $(14/2)^2$
- $x^2 + 14x + 49$

Perfect Square Trinomials

- Create perfect square trinomials.

- $x^2 + 20x + \underline{\hspace{2cm}}$ 100

- $x^2 - 4x + \underline{\hspace{2cm}}$ 4

- $x^2 + 5x + \underline{\hspace{2cm}}$ 25/4



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Solving Quadratic Equations by Completing the Square

Solve the following equation
by **completing the square**:

$$x^2 + 8x - 20 = 0$$

Step 1: Move quadratic
term, and linear term to
left side of the equation

$$x^2 + 8x = 20$$



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Solving Quadratic Equations by Completing the Square

Step 2: Find the term that completes the square on the left side of the equation. Add that term to both sides.

$$x^2 + 8x + \square = 20 + \square$$

$$\frac{1}{2} \cdot (8) = 4 \text{ then square it, } 4^2 = 16$$

$$x^2 + 8x + 16 = 20 + 16$$



Solving Quadratic Equations by Completing the Square

Step 3: Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.

$$x^2 + 8x + 16 = 20 + 16$$

$$(x - 4)(x - 4) = 36$$

$$(x - 4)^2 = 36$$



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Solving Quadratic Equations by Completing the Square

Step 4:

Take the
square root
of each
side

$$\sqrt{(x + 4)^2} = \sqrt{36}$$

$$(x + 4) = \pm 6$$



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Solving Quadratic Equations by Completing the Square

Step 5: Set up the two possibilities and solve

$$x = -4 \pm 6$$

$$x = -4 - 6 \text{ and } x = -4 + 6$$

$$x = -10 \text{ and } x = 2$$

Completing the Square-Example #2

Solve the following equation by **completing the square**:

$$2x^2 - 7x + 12 = 0$$

Step 1: Move quadratic term, and linear term to left side of the equation, the constant to the right side of the equation.

$$2x^2 - 7x = -12$$



Solving Quadratic Equations by Completing the Square

Step 2: Find the term that completes the square on the left side of the equation. Add that term to both sides.

The quadratic coefficient must be equal to 1 before you complete the square, so you must divide all terms by the quadratic coefficient first.

$$2x^2 - 7x + \square = -12 + \square$$

$$\frac{2x^2}{2} - \frac{7x}{2} + \square = -\frac{12}{2} + \square$$

$$x^2 - \frac{7}{2}x + \square = -6 + \square$$

$$\frac{1}{2} \cdot \left(\frac{7}{2}\right) = \frac{7}{4} \text{ then square it, } \left(\frac{7}{4}\right)^2 = \frac{49}{16}$$

$$x^2 - \frac{7}{2}x + \frac{49}{16} = -6 + \frac{49}{16}$$



Solving Quadratic Equations by Completing the Square

Step 3: Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.

$$x^2 - \frac{7}{2}x + \frac{49}{16} = -6 + \frac{49}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = -\frac{96}{16} + \frac{49}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = -\frac{47}{16}$$



Solving Quadratic Equations by Completing the Square

Step 4:

Take the
square
root of
each side

$$\sqrt{\left(x - \frac{7}{4}\right)^2} = \sqrt{\frac{-47}{16}}$$

$$\left(x - \frac{7}{4}\right) = \pm \frac{\sqrt{-47}}{4}$$

$$x = \frac{7}{4} \pm \frac{i\sqrt{47}}{4}$$

$$x = \frac{7 \pm i\sqrt{47}}{4}$$



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Solving Quadratic Equations by Completing the Square

Try the following examples. Do your work on your paper and then check your answers.

1. $x^2 + 2x - 63 = 0$

2. $x^2 + 8x - 84 = 0$

3. $x^2 - 5x - 24 = 0$

4. $x^2 + 7x + 13 = 0$

5. $3x^2 + 5x + 6 = 0$

1. $(-9, 7)$

2. $(6, -14)$

3. $(-3, 8)$

4. $\left(\frac{-7 \pm i\sqrt{3}}{2} \right)$

5. $\left(\frac{-5 \pm i\sqrt{47}}{6} \right)$