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6.5 The Quadratic Formula and the Discriminant

We have a number of different way of finding the roots if a quadratic equations

#1. Making a table

#2. Factoring

#3. Completing the Square

Now a new way that comes from completing the square.

The Quadratic Formula

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The Quadratic Formula

Solve for x by completing the square.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + \ldots = -c + \ldots$$

$$x^2 + \frac{b}{a}x + \ldots = \frac{-c}{a} + \ldots$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

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The Quadratic Formula

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$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

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The Quadratic Formula

Solve for x by completing the square.

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2-4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

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The Quadratic Formula

Solve for x by completing the square.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Yes, you can remember this formula

Pop goes the Weasel

http://www.youtube.com/watch?v=2lbABbfU6Zc&feature=related

Gilligan's Island

http://www.youtube.com/watch?v=3CWTt9QFioY&feature=related

This one I can't explain

http://www.youtube.com/watch?v=haq6kpWdEMs&feature=related

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How does it work

Equation:

$$3x^2 + 5x + 1 = 0$$

$$a = 3$$

$$b = 5$$

$$c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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How does it work

Equation:

$$3x^2 + 5x + 1 = 0$$

$$a = 3$$

$$b = 5$$

$$c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{6}$$

$$x = \frac{-5 \pm \sqrt{13}}{6} = \frac{-5}{6} \pm \frac{\sqrt{13}}{6}$$

The Discriminant

The number in the square root of the quadratic formula.

$$b^2 - 4ac$$

$$Given\ x^2 - 5x + 6 = 0$$

$$(-5)^2 - 4(1)(6)$$

$$25 - 24 = 1$$

The Discriminant

The Discriminant can be negative, positive or zero

If the Discriminant is **positive**,

there are 2 real answers.

If the square root is not a prefect square

(for example), $\sqrt{25}$

then there will be 2 irrational roots

(for example). $2 \pm \sqrt{5}$

The Discriminant

The Discriminant can be negative, positive or zero

If the Discriminant is **positive**,

there are **2 real answers**.

If the Discriminant is **zero**,

there is **1 real answer**.

If the Discriminant is **negative**,

there are **2 complex answers.**

complex answer have i

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Solve using the Quadratic formula

$$x^2 - 8x = 33$$

Solve using the Quadratic formula

$$x^2 - 8x = 33$$

$$x^2 - 8x - 33 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$$

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Solve using the Quadratic formula

$$x^2 - 8x = 33$$

$$x^2 - 8x - 33 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{196}}{2} = \frac{8 \pm 14}{2}$$

$$x = \frac{8+14}{2} = \frac{22}{2} = 11$$

$$x = \frac{8-14}{2} = \frac{-6}{2} = -3$$

Solve using the Quadratic formula

$$x^2 - 34x + 289 = 0$$

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Solve using the Quadratic formula

$$x^2 - 34x + 289 = 0$$

$$x = \frac{-(-34) \pm \sqrt{(-34)^2 - 4(1)(289)}}{2(1)}$$

Solve using the Quadratic formula

$$x^2 - 34x + 289 = 0$$

$$x = \frac{-(-34) \pm \sqrt{(-34)^2 - 4(1)(289)}}{2(1)}$$

$$x = \frac{34 \pm \sqrt{1156 - 1156}}{2}$$

$$x = \frac{34 \pm 0}{2} = \frac{34}{2} = 17$$

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Solve using the Quadratic formula

$$x^2 - 6x + 2 = 0$$

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Solve using the Quadratic formula

$$x^2 - 6x + 2 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 8}}{2} = \frac{6 \pm \sqrt{28}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{7}}{2} = 3 \pm \sqrt{7}$$

Solve using the Quadratic formula

$$x^2 + 13 = 6x$$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

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Solve using the Quadratic formula

$$x^2+13=6x$$

$$x^2-6x+13=0$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-52}}{2}=\frac{6\pm\sqrt{-16}}{2}$$

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Solve using the Quadratic formula

$$x^2 + 13 = 6x$$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm 4i}{2} = \frac{6}{2} \pm \frac{4}{2}i$$

$$x = 3 \pm 2i$$

Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

0, One rational root

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Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

0, One rational root

$$x^2 + 3x + 5 = 0$$

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Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

0, One rational root

$$x^2 + 3x + 5 = 0$$

-11, Two complex roots

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Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

0, One rational root

$$x^2 + 3x + 5 = 0$$

-11, Two complex roots

$$x^2 + 8x - 4 = 0$$

Describe the roots

Tell me the Discriminant and the type of roots

$$x^2 + 6x + 9 = 0$$

0, One rational root

$$x^2 + 3x + 5 = 0$$

-11, Two complex roots

$$x^2 + 8x - 4 = 0$$

80, Two irrational roots