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HIGH SCHOOL

Solving Systems of Linear and Quadratic Equations



Remember these?

$$2x + y = 8$$

$$x - y = 10$$

Systems of Linear Equations

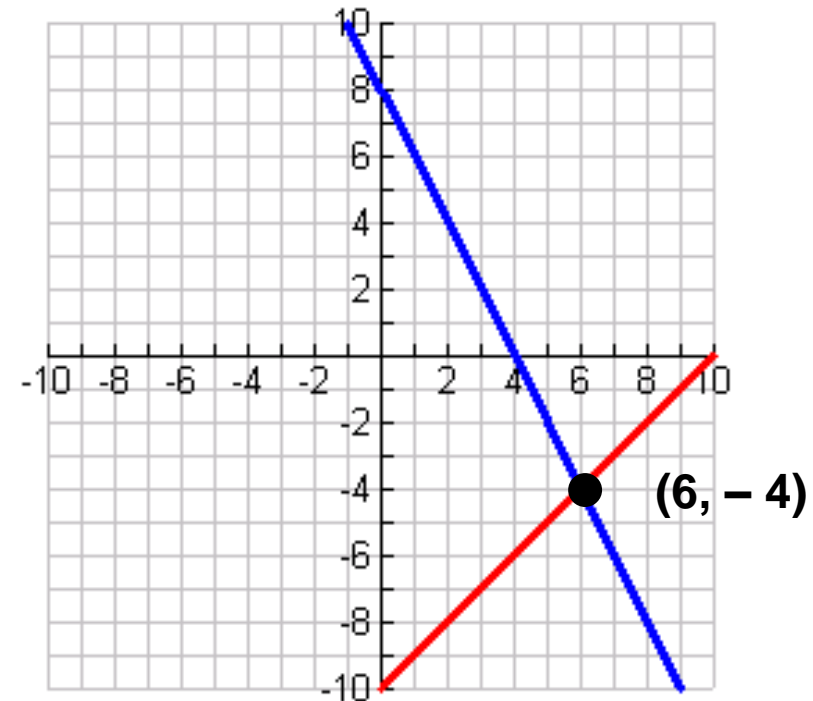
We had 3 methods to solve them.

Method 1 - Graphing

Solve for y.

$$y = -2x + 8$$

$$y = x - 10$$





Remember these?

Systems of Linear Equations

$$2x + y = 8$$

$$x - y = 10$$

Method 2 - Substitution

$$y = -2x + 8$$

$$x - (-y = 10) = 10$$

$$x + 2x - 8 = 10$$

$$3x = 18$$

$$x = 6$$

$$y = -2(2x + 8) + 8$$

$$y = -12 + 8$$

$$y = -4$$

$$(6, -4)$$



Remember these?

Systems of Linear Equations

$$2x + y = 8$$

$$x - y = 10$$

Method 3 - Elimination

$$\begin{array}{r} 2x + y = 8 \\ + \quad x - y = 10 \\ \hline 3x = 18 \\ x = 6 \end{array}$$

$$\left(\begin{array}{r} x - y = 10 \\ -y = 4 \end{array} \right.$$

$$-y = 4$$

$$y = -4$$

$$(6, -4)$$

**All 3 methods
giving us the
same answer
(6, -4).**



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Try the following.

$$1. \begin{cases} x + 3y = -9 \\ 4y = x + 16 \end{cases} \quad (12, 1)$$

$$2. \begin{cases} 2x + 3y = 3 \\ -6x - 12y = -11 \end{cases} \quad \left(\frac{1}{2}, \frac{2}{3}\right)$$

$$3. \begin{cases} 3m + 6n = 12 \\ 4m + 5n = 28 \end{cases} \quad (12, -4)$$

$$4. \begin{cases} -3x + 6y = 18 \\ y = \frac{1}{2}x + 3 \end{cases} \quad \begin{array}{l} \text{Many solutions} \\ \text{Same Line!} \end{array}$$

$$5. \begin{cases} 5x - \frac{28}{3}y = 2 \\ -3x = 4y \end{cases} \quad \left(\frac{1}{6}, -\frac{1}{8}\right)$$

$$6. \begin{cases} \frac{1}{2}x - \frac{1}{3}y = \frac{1}{6} \\ 6x - 4y = 1 \end{cases} \quad \begin{array}{l} \text{NO solution} \\ \text{Parallel Lines!} \end{array}$$



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**Now let's look at
systems of
Linear and Quadratic
Equations!**



Solve the System Algebraically

Use Substitution

$$y = x^2 + 1$$

$$y - x = 1$$

$$(x^2 + 1) - x = 1$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 0 \quad \text{or} \quad x = 1$$

$$\boxed{x = 0}$$

$$y = 0^2 + 1 \quad \mathbf{(0, 1)}$$

$$y = 1$$

$$\boxed{x = 1}$$

$$y = 1^2 + 1 \quad \mathbf{(1, 2)}$$

$$y = 2$$

Answer: $(0, 1)$ $(1, 2)$



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Solve the System Algebraically

Use Substitution

$$y = x^2 - 2x + 2$$

$$y - 2x = -2$$

$$y = 2x - 2$$

$$\boxed{x = 2}$$

$$(2x - 2) = x^2 - 2x + 2$$

$$y - 2(2) = -2 \quad \mathbf{(2, 2)}$$

$$0 = x^2 - 4x + 4$$

$$y - 4 = -2$$

$$0 = (x - 2)(x - 2)$$

$$y = 2$$

$$0 = x - 2$$

$$x = 2$$

Answer: $\mathbf{(2, 2)}$



Solve the System Algebraically

Use Substitution

$$x^2 + y^2 = 26$$

$$x - y = 6$$

$$y = x - 6$$

$$x^2 + (x - 6)^2 = 26$$

$$x^2 + x^2 - 12x + 36 = 26$$

$$2x^2 - 12x + 10 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

$$x = 5 \text{ or } x = 1$$

$$\boxed{x = 5}$$

$$5 - y = 6$$

$$-y = 1$$

$$y = -1$$

$$(5, -1)$$

$$\boxed{x = 1}$$

$$1 - y = 6$$

$$-y = 5$$

$$y = -5$$

$$(1, -5)$$

Answer: $(5, -1)$ $(1, -5)$



Solve the System Algebraically

Use Substitution

$$x^2 + y^2 = 25$$

$$4y = 3x$$

$$y = \frac{3}{4}x$$

$$x^2 + \left(\frac{3}{4}x\right)^2 = 25$$

$$x^2 + \frac{9}{16}x^2 = 25$$

$$16x^2 + 9x^2 = 400$$

$$25x^2 = 400$$

$$x^2 = 16$$

$$x = \pm 4$$

$$\boxed{x = 4}$$

$$4y = 3(4)$$

(4, 3)

$$4y = 12$$

$$y = 3$$

$$\boxed{x = -4}$$

$$4y = 3(-4)$$

(-4, -3)

$$4y = -12$$

$$y = -3$$

Answer: **(4, 3) (-4, -3)**



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Solve the System Algebraically

Use ????

$$3x^2 - 4y^2 = 11$$

$$x^2 + y^2 = 13$$



Now let's look at the Graphs of these Systems!

$$y = x^2 - 4$$

$$y = -x - 2$$

What does the graph of each look like?

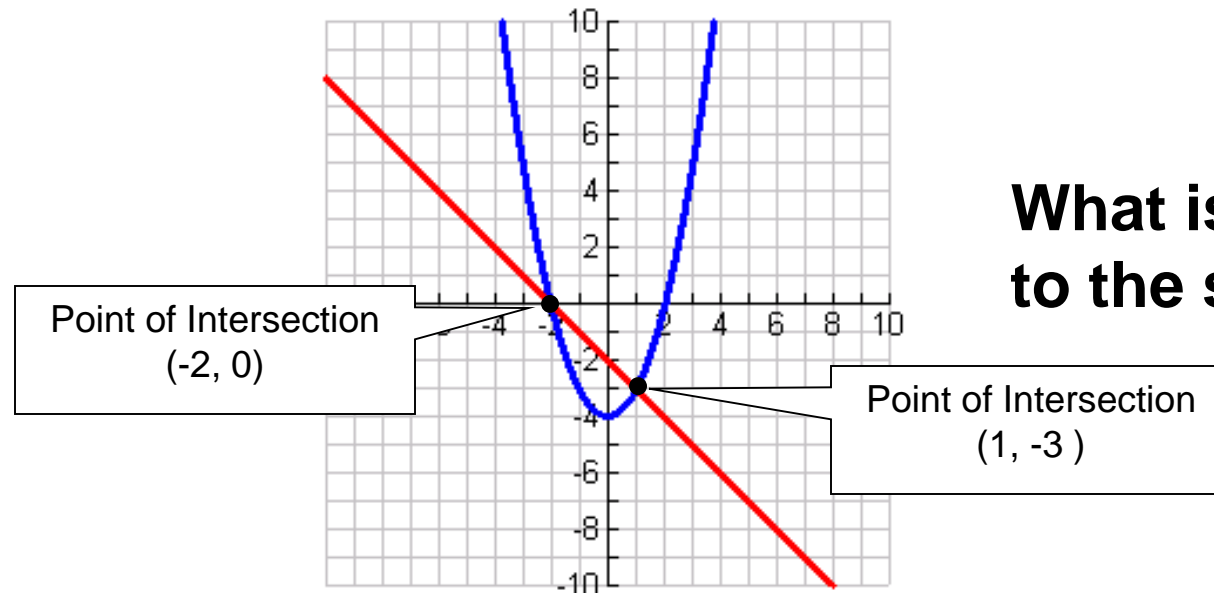
Quadratic

Parabola

Classify each equation as linear/quadratic.

Linear

Line



What is the solution to the system?



We have solved the following algebraically

Now use your calculator
to check it graphically.

$$y = x^2 + 1$$

$$y - x = 1$$

Answer:

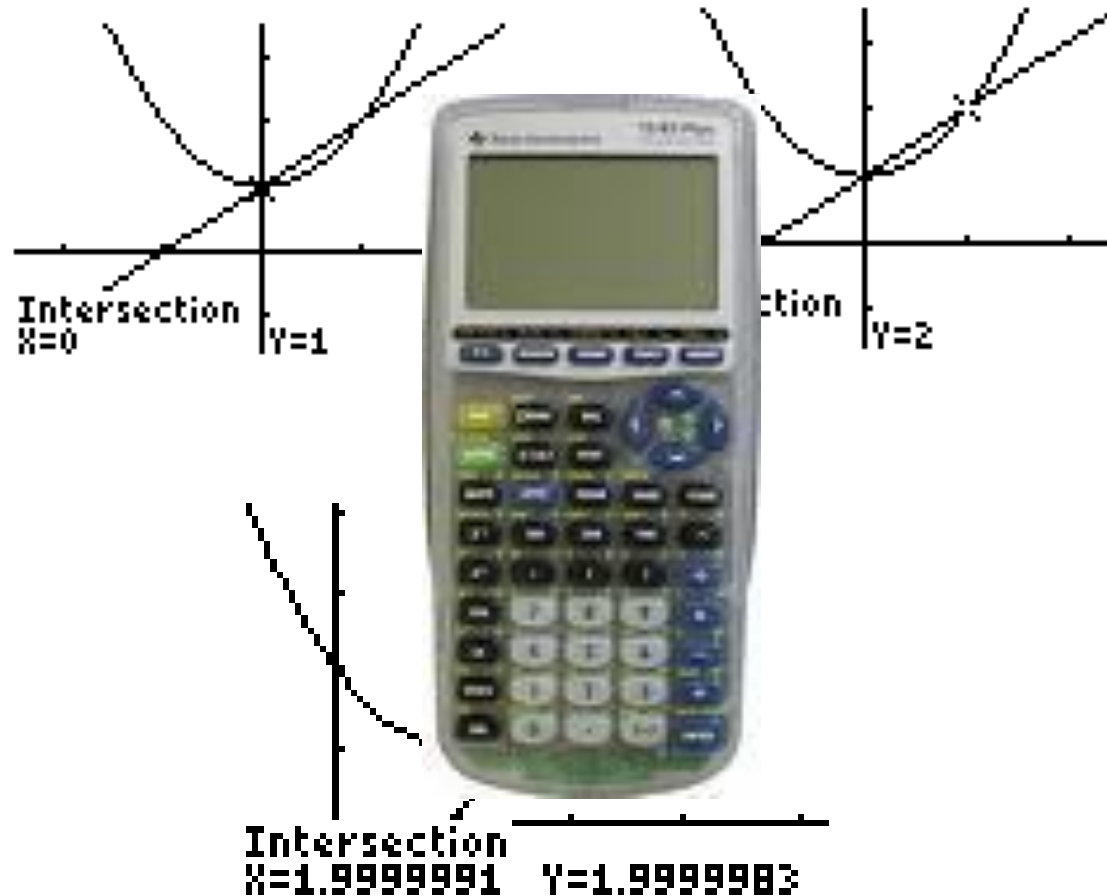
$(0,1) (1,2)$

$$y = x^2 - 2x + 2$$

$$y - 2x = -2$$

Answer:

$(2,2)$





Equations must be solved for y!!

$$x^2 + y^2 = 26$$

$$x - y = 6$$

$$x^2 + y^2 = 26$$

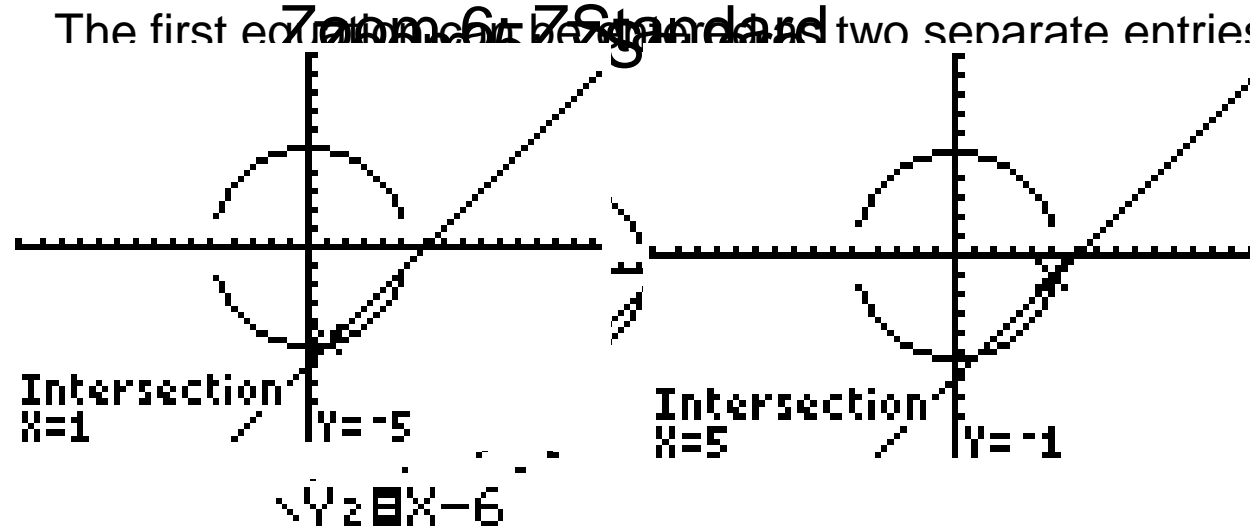
$$y^2 = 26 - x^2$$

$$y = \pm\sqrt{26 - x^2}$$

and

$$y = x - 6$$

The first equation can be graphed as two separate entries:



Answer: **(5, -1) (1, -5)**