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Rational Expressions

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Multiplying / Dividing

- **Let's first review how we multiply and divide fractions.**

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Multiplying / Dividing

- **When multiplying/ dividing, do we have to have a common denominator?**
- **Nope**

Multiplying / Dividing

- **Is there anything special that we have to do?**
- **Nope**

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Multiplying / Dividing

- **How do we multiply fractions?**
- **Multiply numerators and denominators.**

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Multiplying / Dividing

- **Examples:**

$$\frac{4x}{3y} \bullet \frac{5x}{-3} = ?$$

$$\frac{9a}{(b-3c)} \bullet \frac{a+3}{4} = ?$$

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Multiplying / Dividing

- **Examples:**

$$\frac{4x}{3y} \bullet \frac{5x}{-3} = \frac{20x^2}{-9y}$$

$$\frac{9a}{(b-3c)} \bullet \frac{a+3}{4} = \frac{9a^2+27a}{4b-12c}$$

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Multiplying / Dividing

- **Let's talk about simplifying.**
- **When can we cancel things in the numerator and denominator?**

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Multiplying / Dividing

- **You must have exactly the same factor.**
- **x - 3 can only be simplified by x - 3, not by x, not by 3, only by x - 3.**

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Multiplying / Dividing

- **Simplify:**

$$\frac{3(2x-7)(x+5)}{9(x+5)(7x-2)} = ?$$

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Multiplying / Dividing

- **Simplify:**

$$\frac{3(2x-7)(x+5)}{9(x+5)(7x-2)} =$$

$$\frac{2x-7}{3(7x-2)}$$

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Multiplying / Dividing

- **Simplify:**

$$\frac{2x+10}{x+5}$$

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Multiplying / Dividing

- **Simplify: factor first**

$$\frac{2x+10}{x+5} = \frac{2(x+5)}{x+5}$$

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- **Simplify: cancel**

$$\frac{2x+10}{x+5} = \frac{2(x+5)}{x+5} = 2$$

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Multiplying / Dividing

- **Simplify: factor, then cancel**

$$\frac{6b^3 - 24b^2}{b^2 + b - 20}$$

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Multiplying / Dividing

- **Simplify: factor, then cancel**

$$\frac{6b^3 - 24b^2}{b^2 + b - 20} =$$

$$\frac{6b^2(b-4)}{(b+5)(b-4)} = \frac{6b^2}{b+5}$$

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Multiplying / Dividing

- **When multiplying rational expressions, factor each numerator and denominator first, then cancel, then multiply (squish them together).**

Multiplying / Dividing

- **Steps to multiply:**
- **1)factor completely**
- **2)cancel**
- **3)multiply (squish)**

Multiplying / Dividing

- **Multiply:** $\dfrac{x^3}{2y^2} \bullet \dfrac{6y^4}{xy}$
- **We have nothing to factor so just cancel and multiply.**

Multiplying / Dividing

- **Multiply:**

$$\frac{x^3}{2y^2} \bullet \frac{6y^4}{xy} = \frac{3x^2y}{1} = 3x^2y$$

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Multiplying / Dividing

- **Multiply:**

$$\frac{2x^2+5x-7}{x+4} \bullet \frac{x^2+4x}{x^2-2x+1}$$

Multiplying / Dividing

- **Factor:**

$$\frac{(2x+7)(x-1)}{x+4} \bullet \frac{x(x+4)}{(x-1)(x-1)}$$

Multiplying / Dividing

- **Cancel:**

$$\frac{(2x+7)(x-1)}{x+4} \bullet \frac{x(x+4)}{(x-1)(x-1)}$$

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Multiplying / Dividing

- **Multiply:**

$$\frac{(2x+7)}{1} \bullet \frac{x}{(x-1)} = \frac{x(2x+7)}{x-1}$$

Multiplying / Dividing

- **Realize: sometimes you may see people go ahead and multiply (such as the last numerator), you don't have to do this for me. Just be able to recognize it if you see it.**

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Multiplying / Dividing

- **Now on to dividing.**
- **This is exactly like multiplying, except for ONE step.**

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Multiplying / Dividing

- **How do we divide fractions?**

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \bullet \frac{4}{1} = \frac{12}{4} = 3$$

- **We multiply by the reciprocal (inverse) of the divisor (2nd fraction).**

Multiplying / Dividing

- **Divide:**

$$\frac{x^2 - x - 12}{x^2 + 11x + 24} \div \frac{x^2 - 2x - 8}{x^2 + 8x}$$

Multiplying / Dividing

- **Divide: first step is flip & multiply.**

$$\frac{x^2 - x - 12}{x^2 + 11x + 24} \bullet \frac{x^2 + 8x}{x^2 - 2x - 8}$$

Multiplying / Dividing

- **Divide: Now proceed like a multiplication problem. Factor first, cancel, multiply.**

$$\frac{(x-4)(x+3)}{(x+3)(x+8)} \bullet \frac{x(x+8)}{(x-4)(x+2)}$$

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Multiplying / Dividing

$$\frac{(x-4)(x+3)}{(x+3)(x+8)} \bullet \frac{x(x+8)}{(x-4)(x+2)}$$

$$\frac{(x-4)(x+3)}{(x+3)(x+8)} \bullet \frac{x(x+8)}{(x-4)(x+2)}$$

Multiplying / Dividing

- **Simplify**

$$\frac{x}{(x+2)}$$

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Adding/Subtracting

- **What do we have to have in order to add or subtract fractions?**
- **Right a common denominator.**

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Adding/Subtracting

- **when we talk about CDs, we mean denominators that contain the same factors.**
- **To find our CD, we will first factor the ones we have.**

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Adding/Subtracting

- **Then we will multiply each denominator by the factors it is missing to create a CD.**
- **Remember, we must also multiply the numerator by that same factor.**

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Adding/Subtracting

- **Find the common denominator for these two rational expressions.**

$$\frac{2x}{5ab^3} + \frac{4y}{3a^2b^2}$$

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Adding/Subtracting

$$\frac{2x}{5ab^3} + \frac{4y}{3a^2b^2} =$$

$$\frac{2x(3a)}{5ab^3(3a)} + \frac{4y(5b)}{3a^2b^2(5b)}$$

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Adding/Subtracting

$$\frac{2x(3a)}{5ab^3(3a)} + \frac{4y(5b)}{3a^2b^2(5b)} =$$

$$\frac{6ax}{15a^2b^3} + \frac{20by}{15a^2b^3}$$

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Adding/Subtracting

- **Now that we have a CD, we just simplify (we did already) and add the numerators.**

$$\frac{6ax + 20by}{15a^2b^3}$$

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Adding/Subtracting

- **If we could factor the numerator and denominator, we would to see if we could simplify more.**

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Steps to Add/Subtract

- **Factor denominators**
- **Find CD & equivalent fractions**
- **Simplify/add/sub numer.**
- **Factor numer/denom**
- **Cancel**

Adding/Subtracting

- **Simplify:**

$$\frac{x}{x^2+5x+6} - \frac{2}{x^2+4x+4}$$

Adding/Subtracting

- **Factor**

$$\frac{x}{(x+2)(x+3)} - \frac{2}{(x+2)(x+2)}$$

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Adding/Subtracting

- **Find CD**
- **We need a factor of (x+2) in the first denominator and a factor of (x+3) in the second denominator.**

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Adding/Subtracting

- **Create equivalent fractions.**

$$\frac{x(x+2)}{(x+2)^2(x+3)} - \frac{2(x+3)}{(x+2)^2(x+3)}$$

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Adding/Subtracting

- **Simplify the numerators.**

$$\frac{x^2+2x}{(x+2)^2(x+3)} - \frac{2x+6}{(x+2)^2(x+3)}$$

Adding/Subtracting

- **Subtract.**
- **$(x^2 + 2x) - (2x + 6) = x^2 - 6$**

$$\frac{x^2 - 6}{(x+2)^2(x+3)}$$

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Adding/Subtracting

- **Can we factor the numerator? No, we are done.**

$$\frac{x^2 - 6}{(x+2)^2(x+3)}$$

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Adding/Subtracting

- **Simplify:**

$$\frac{x-5}{2x-6} - \frac{x-7}{4x-12}$$

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Adding/Subtracting

$$\frac{x-5}{2(x-3)} - \frac{x-7}{4(x-3)} =$$

$$\frac{2(x-5)}{4(x-3)} - \frac{x-7}{4(x-3)} =$$

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Adding/Subtracting

$$\frac{2x-10}{4(x-3)} - \frac{x-7}{4(x-3)} =$$

$$\frac{x-3}{4(x-3)} = \frac{1}{4}$$

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Complex Fractions

- **Complex fractions are those fractions whose numerator & denominator both contain fractions.**
- **To simplify them, we just multiply by the CD.**

Complex Fractions

- **Example:**
- **What would be the CD?**
- **6**
- **Multiply every term by 6.**

$$\frac{x+\frac{x}{3}}{x-\frac{x}{6}}$$

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Complex Fractions

- **Simplify.**

$$\frac{6x+6\left(\frac{x}{3}\right)}{6x-6\left(\frac{x}{6}\right)}=\frac{6x+2x}{6x-x}$$

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Complex Fractions

- **Simplify.**

$$\frac{6x+2x}{6x-x} = \frac{8x}{5x} = \frac{8}{5}$$

Complex Fractions

- **Simplify.**
- **CD is ?**
- **xy**

$$\frac{\frac{2x}{y}+1}{\frac{2x}{y}+\frac{y}{x}}$$

Complex Fractions

- **Multiply each term by xy.**

$$\frac{xy\left(\frac{2x}{y}\right)+1(xy)}{(xy)\left(\frac{2x}{y}\right)+(xy)\left(\frac{y}{x}\right)}$$

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Complex Fractions

- **Simplfy.**

$$\frac{x\cancel{y}\left(\frac{2x}{\cancel{y}}\right)+1(xy)}{(x\cancel{y})\left(\frac{2x}{\cancel{y}}\right)+(\cancel{x}y)\left(\frac{y}{\cancel{x}}\right)}$$

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Complex Fractions

- **Final answer.**

$$\frac{2x^2 + xy}{2x^2 + y^2}$$

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Complex Fractions

- **Steps to simplify.**
- **Find the CD**
- **Multiply EACH term by the CD**
- **Cancel and Simplify**

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Complex Fractions

- **Work on Complex Fraction Worksheet**

Rational Equations

- **Solving Rational Equations would be easy, except for the rational part.**
- **How can we get rid of the the fractions?**

Rational Equations

- **Multiply BOTH SIDES by the common denominator**
- **This will be very similar to adding/subtracting rational expressions.**

Rational Equations

- **Once you have multiplied by the CD, just solve the equation like you would normally. Use your calculator if you want to.**

Rational Equations

- **Solve:**

$$\frac{24}{r-3} = \frac{36}{r+3}$$

Rational Equations

- **What is the CD?**
- **(r - 3)(r + 3)**

$$\frac{24}{r-3} = \frac{36}{r+3}$$

Rational Equations

- **Multiply by (r - 3)(r + 3) on both sides.**

$$\frac{24(r+3)(r-3)}{r-3} = \frac{36(r+3)(r-3)}{r+3}$$

Rational Equations

- **Simplify & distribute**

$$\frac{24(r+3)(r-3)}{r-3} = \frac{36(r+3)(r-3)}{r+3}$$

$$24(r+3) = 36(r-3)$$

$$24r + 72 = 36r - 108$$

Rational Equations

- **Solve**

$$24r + 72 = 36r - 108$$

$$180 = 12r$$

$$15 = r$$

Rational Equations

- **Solve:**

$$\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$$

Rational Equations

- **Common Denominator is**
- **6(x-2)**

$$\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$$

Rational Equations

- **Multiply by 6(x-2)**

$$\frac{6(x-2)(x+1)}{3(x-2)} =$$

$$\frac{(5x)(6)(x-2)}{6} + \frac{1(6)(x-2)}{x-2}$$

Rational Equations

- **Cancel**

$$2\frac{\cancel{6}(\cancel{x-2})(x+1)}{\cancel{3}(\cancel{x-2})} = \frac{(5x)(\cancel{6})(x-2)}{\cancel{6}} + \frac{1(6)(\cancel{x-2})}{\cancel{x-2}}$$

Rational Equations

- **Simplify**

$$2(x+1)=5x(x-2)+6$$

$$2x+2=5x^2-10x+6$$

$$0=5x^2-2x-10x+6-2$$

$$5x^2-12x+4=0$$

Rational Equations

- **Solve**

$$5x^2 - 12x + 4 = 0$$

$$5x^2 - 10x - 2x + 4 = 0$$

$$(5x^2 - 10x) + (-2x + 4) = 0$$

Rational Equations

- **Solve**

$$5x(x-2)+-2(x-2)=0$$

$$(5x-2)(x-2)=0$$

$$x=\frac{2}{5},2$$