



James Madison
HIGH SCHOOL

Permutations and Combinations

Fundamental Counting Principle

Fundamental Counting Principle can be used to determine the number of possible outcomes when there are two or more characteristics.

Fundamental Counting Principle states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m * n$ possible outcomes for the two events together.

Fundamental Counting Principle

Lets start with a simple example.

A student is to roll a die and flip a coin.
How many possible outcomes will there be?

1H	2H	3H	4H	5H	6H	$6*2 = 12$ outcomes
1T	2T	3T	4T	5T	6T	

12 outcomes

Fundamental Counting Principle

For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

$$4*3*2*5 = 120 \text{ outfits}$$

Permutations

A **Permutation** is an arrangement of items in a particular order.

Notice, **ORDER MATTERS!**

To find the number of Permutations of n items, we can use the Fundamental Counting Principle or factorial notation.

Permutations

The number of ways to arrange
the letters *ABC*:

	_____	_____	_____
Number of choices for first blank?	3	_____	_____
Number of choices for second blank?	3	2	_____
Number of choices for third blank?	3	2	1

$$3 * 2 * 1 = 6 \qquad 3! = 3 * 2 * 1 = 6$$

ABC ACB BAC BCA CAB CBA

Permutations

To find the number of Permutations of n items chosen r at a time, you can use the formula

$${}_n P_r = \frac{n!}{(n-r)!} \quad \text{where } 0 \leq r \leq n.$$

$${}_5 P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 * 4 * 3 = 60$$

Permutations

Practice:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

[Answer Now](#)

Permutations

Practice:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

$${}_{30}P_3 = \frac{30!}{(30-3)!} = \frac{30!}{27!} = 30 * 29 * 28 = 24360$$

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

[Answer Now](#)

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

$${}_{24}P_5 = \frac{24!}{(24-5)!} = \frac{24!}{19!} =$$

$$24 * 23 * 22 * 21 * 20 = 5,100,480$$

Combinations

A **Combination** is an arrangement of items in which order does not matter.

ORDER DOES NOT MATTER!

Since the order does not matter in combinations, there are fewer combinations than permutations. The combinations are a "subset" of the permutations.

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$${}^n C_r = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \leq r \leq n.$$

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$${}_n C_r = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \leq r \leq n.$$

$${}_5 C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} =$$

$$\frac{5 * 4 * 3 * 2 * 1}{3 * 2 * 1 * 2 * 1} = \frac{5 * 4}{2 * 1} = \frac{20}{2} = 10$$

Combinations

Practice:

To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

[Answer Now](#)



Combinations

Practice: To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

$${}_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} =$$

$$\frac{52 * 51 * 50 * 49 * 48}{5 * 4 * 3 * 2 * 1} = 2,598,960$$

Combinations

Practice:

A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

[Answer Now](#)



Combinations

Practice: A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

$${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5*4}{2*1} = 10$$

Combinations

Practice:

A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

[Answer Now](#)



Combinations

Practice: A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

$$\begin{array}{ccc} \text{Center:} & \text{Forwards:} & \text{Guards:} \\ {}_2C_1 = \frac{2!}{1!1!} = 2 & {}_5C_2 = \frac{5!}{2!3!} = \frac{5*4}{2*1} = 10 & {}_4C_2 = \frac{4!}{2!2!} = \frac{4*3}{2*1} = 6 \\ & {}_2C_1 * {}_5C_2 * {}_4C_2 & \end{array}$$

Thus, the number of ways to select the starting line up is $2*10*6 = 120$.