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Patterns and Inductive Reasoning

- Inductive reasoning – reasoning based on patterns you observe.
 - You can observe patterns in some number sequences and some sequences of geometric figures to discover relationships.

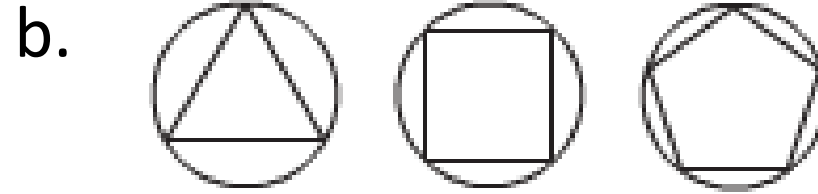


James Madison Finding and Using a Pattern

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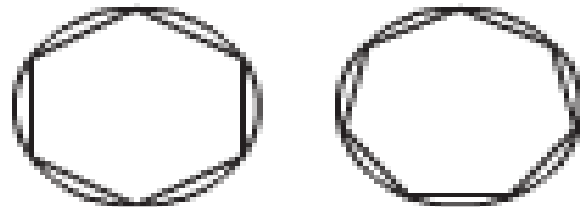
- Look for a pattern. What are the next two terms in each sequence?

a. 3, 9, 27, 81, ...



a. Each term is three times the previous term. The next two terms are $81 * 3 = 243$ and $243 * 3 = 729$.

b. Each circle contains a polygon that has one more side than the preceding polygon. The next two circles contain a six-sided and a seven-sided polygon.





James Madison HIGH SCHOOL Conjecture

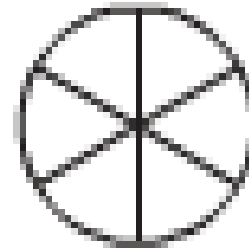
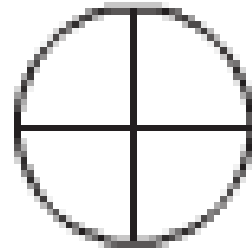
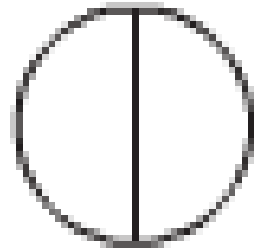
- You may want to find the tenth or the one-hundredth term in a sequence.
 - Rather than find every previous term, you look for a pattern and make a conjecture.
- A **conjecture** – a conclusion you reach using inductive reasoning.



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Using Inductive Reasoning

- Look at the circles. What conjecture can you make about the number of regions 20 diameters form?



- Each c
diame

ters. Twenty



Collecting Information to Make a Conjecture

- What conjecture can you make about the sum of the first 30 even numbers?
- Find the first few sums and look for a pattern.

Number of Terms	Sum
1	$2 = 2 = 1 \cdot 2$
2	$2 + 4 = 6 = 2 \cdot 3$
3	$2 + 4 + 6 = 12 = 3 \cdot 4$
4	$2 + 4 + 6 + 8 = 20 = 4 \cdot 5$

- You can see that the sum of the first n even numbers is $n \cdot (n + 1)$. So the sum of the first 30 even numbers is $30 \cdot 31 = 930$.



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Counterexample

- Not all conjectures turn out to be true.
 - You should test your conjecture many times.
 - You can prove a conjecture is false by finding *one* counterexample.

- A **counterexample** – an example that shows that a conjecture is incorrect.



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Finding a Counterexample

- What is a counterexample for each conjecture?
 - A. If the name of a month starts with the letter J, it is a summer month.
 - B. You can connect any three points to form a triangle.
 - C. When you multiply a number by 2, the product is greater than the original number.

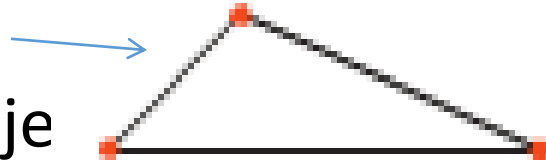


Finding a Counterexample

A. Counterexample: January starts with J and it is a winter month.

B. Counterexample: if the three points lie on a line, you cannot form a triangle.

Support



Counterexample



C. The conjecture is true for positive numbers and zero.

but it is false

Counterexample: $-4 * 2 = -8$ and -8 is not greater than $-$

4.