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Triangle Inequalities

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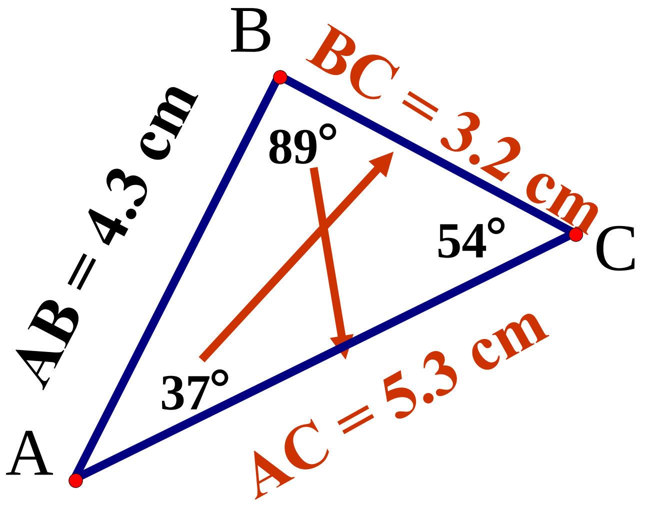
Triangle Inequality

- The smallest side is across from the smallest angle.

  $\angle A$ is the smallest angle, $\therefore \overline{BC}$ is the smallest side.

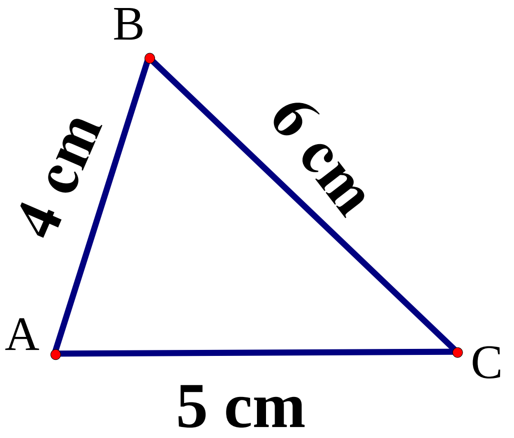
- The largest angle is across from the largest side.

  $\angle B$ is the largest angle, $\therefore \overline{AC}$ is the largest side.



Triangle Inequality – *examples*…

For the triangle, list the angles in order from least to greatest measure.



$\overline{AB}$ *is the smallest side* $\rightarrow \angle C$ *smallest angle.*

$\overline{BC}$ *is the largest side* $\rightarrow \angle A$ *is the largest angle.*

Angles in order from least to greatest $\rightarrow \angle C, \angle B, \angle A$

Triangle Inequality – *examples*…

For the triangle, list the sides in order from shortest to longest measure.

$(7x + 8)° + (7x + 6)° + (8x – 10)° = 180°$

$22x + 4 = 180°$

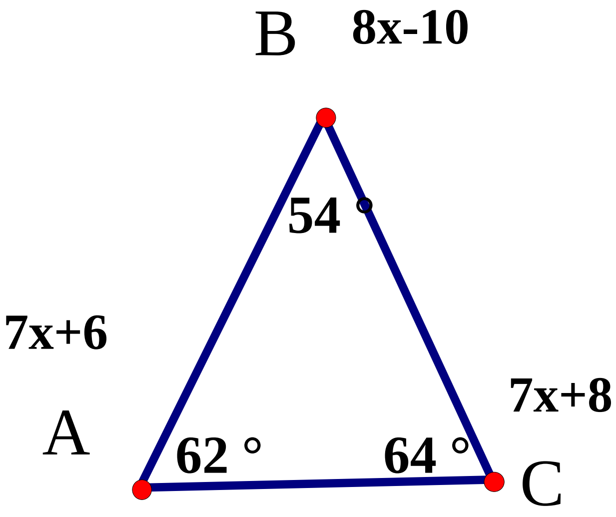
$22x = 176$

$X = 8$

m<C = 7x + 8 = 64 °

m<A = 7x + 6 = 62 °

m<B = 8x – 10 = 54 °



$\angle B$ *is the smallest angle* $\rightarrow \overline{AC}$ *shortest side.*

$\angle C$ *is the largest angle* $\rightarrow \overline{AB}$ *is the longest side.*

Sides in order from smallest to longest $\rightarrow \overline{AC}, \overline{BC}, \overline{AB}$

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Converse Theorem & Corollaries

Converse: If one angle of a triangle is larger than a second angle, then the side opposite the first angle is larger than the side opposite the second angle.

Corollary 1: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Corollary 2: The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

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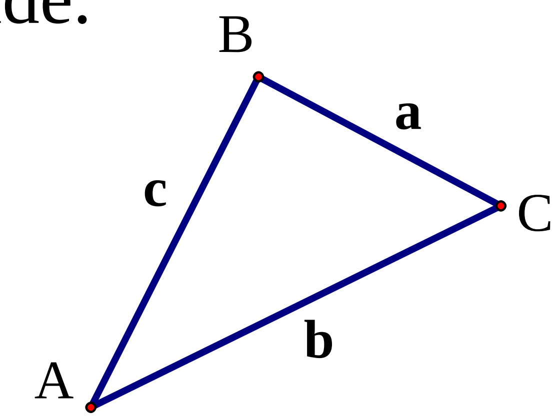
Triangle Inequality Theorem:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

a + b > c

a + c > b

b + c > a



Example: Determine if it is possible to draw a triangle with side measures 12, 11, and 17.

12 + 11 > 17 → Yes

11 + 17 > 12 → Yes

12 + 17 > 11 → Yes

Therefore a triangle can be drawn.

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Finding the range of the third side:

Since the third side cannot be larger than the other two added together, we find the **maximum** value by **adding** the two sides.

Since the third side and the smallest side cannot be larger than the other side, we find the **minimum** value by **subtracting** the two sides.

Example: Given a triangle with sides of length 3 and 8, find the range of possible values for the third side.

The maximum value (if x is the largest side of the triangle)	The minimum value (if x is not that largest side of the Δ)
$3 + 8 > x$	$8 – 3 > x$
$11 > x$	$5 > x$

Range of the third side is $5 < x < 11$.