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Vocabulary

coordinate proof

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You have used coordinate geometry to find the midpoint of a line segment and to find the distance between two points. Coordinate geometry can also be used to prove conjectures.

A **coordinate proof** is a style of proof that uses coordinate geometry and algebra. The first step of a coordinate proof is to position the given figure in the plane. You can use any position, but some strategies can make the steps of the proof simpler.

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Strategies for Positioning Figures in the Coordinate Plane

- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

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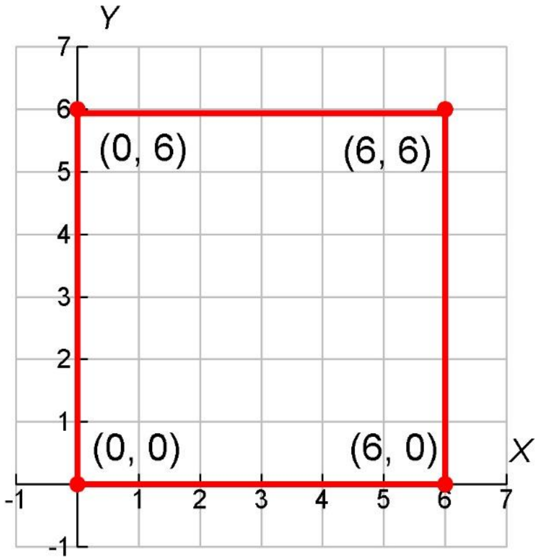
Example 1: Positioning a Figure in the Coordinate Plane

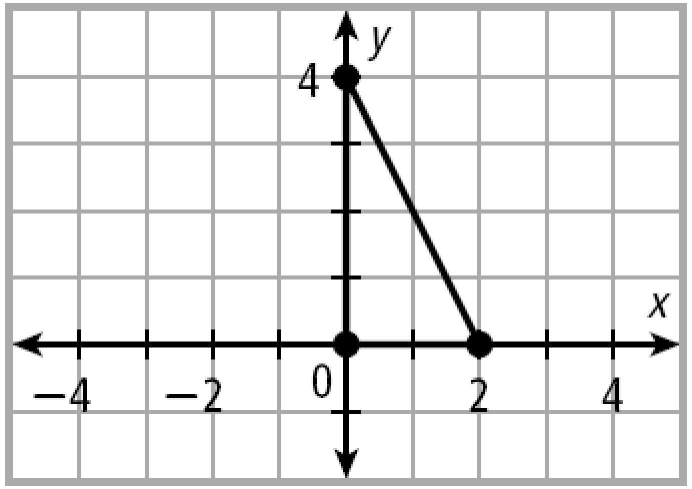
Position a square with a side length of 6 units in the coordinate plane.

You can put one corner of the square at the origin.



Check It Out! Example 1

Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane. (*Hint*: Use the origin as the vertex of the right angle.)



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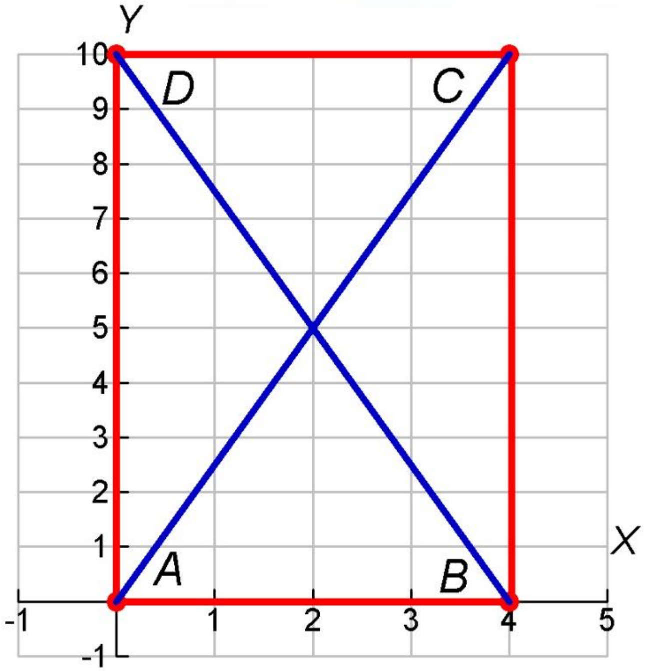
Once the figure is placed in the coordinate plane, you can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

Example 2: Writing a Proof Using Coordinate Geometry

Write a coordinate proof.

Given: Rectangle *ABCD* with *A*(0, 0), *B*(4, 0), *C*(4, 10), and *D*(0, 10)

Prove: The diagonals bisect each other.



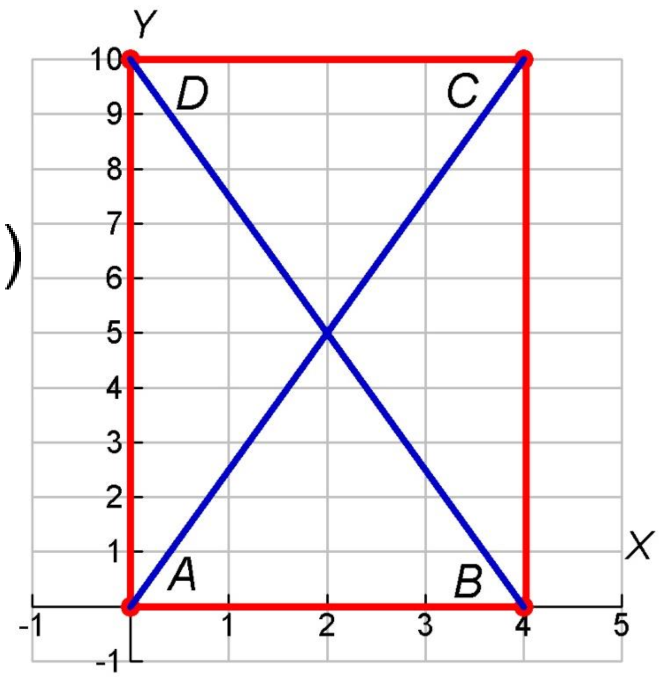
Example 2 Continued

By the Midpoint Formula,

$$\text{mdpt. of } \overline{AC} = \left(\frac{0+4}{2}, \frac{0+10}{2}\right) = (2,5)$$

$$\text{mdpt. of } \overline{BD} = \left(\frac{4+0}{2}, \frac{0+10}{2}\right) = (2,5)$$

The midpoints coincide, therefore the diagonals bisect each other.



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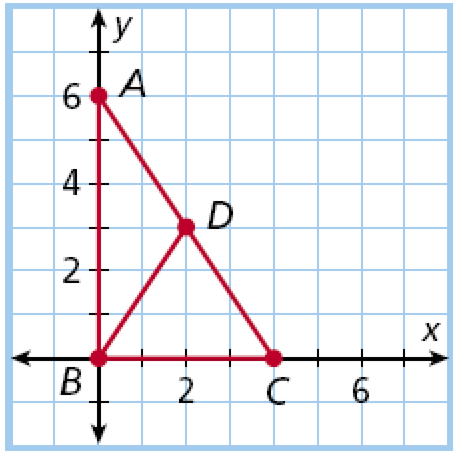
Check It Out! Example 2

Use the information in Example 2 to write a coordinate proof showing that the area of ∆*ADB* is one half the area of ∆*ABC*.

Proof: ∆*ABC* is a right triangle with height *AB* and base *BC*.



$$\text{area of } \Delta ABC = \frac{1}{2}bh$$

$$= \frac{1}{2}(4)(6) = 12 \text{ square units}$$

Check It Out! Example 2 Continued

By the Midpoint Formula, the coordinates of

$$D = \left(\frac{0+4}{2}, \frac{6+0}{2}\right) = (2, 3).$$

The *x*-coordinate of *D* is the height of Δ*ADB*, and the base is 6 units.

$$\text{The area of } \Delta ADB = \frac{1}{2}bh$$

$$= \frac{1}{2}(6)(2) = 6 \text{ square units}$$

Since $6 = \frac{1}{2}(12)$, the area of Δ*ADB* is one half the area of Δ*ABC*.

A coordinate proof can also be used to prove that a certain relationship is always true.

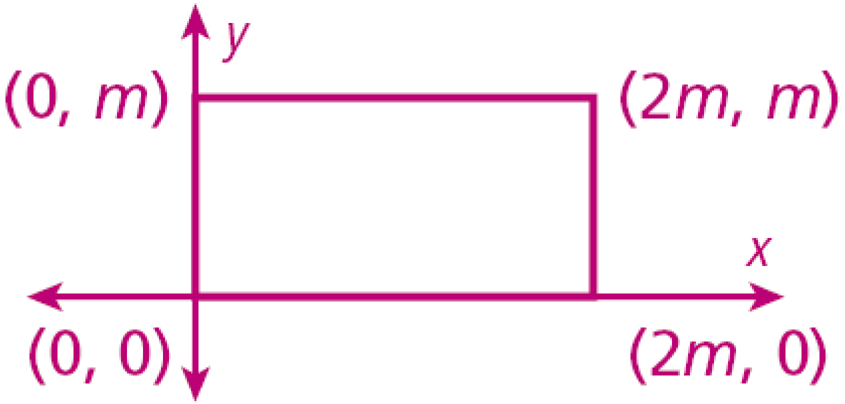
You can prove that a statement is true for all right triangles without knowing the side lengths.

To do this, assign variables as the coordinates of the vertices.

Example 3A: Assigning Coordinates to Vertices

Position each figure in the coordinate plane and give the coordinates of each vertex.

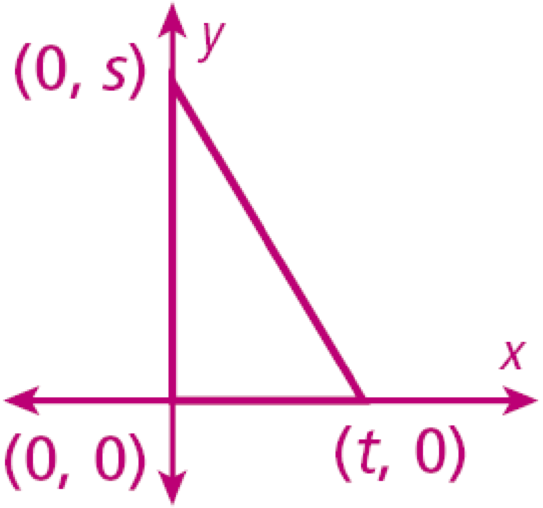
rectangle with width *m* and length twice the width



Example 3B: Assigning Coordinates to Vertices

Position each figure in the coordinate plane and give the coordinates of each vertex.

right triangle with legs of lengths *s* and *t*



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Caution!

Do not use both axes when positioning a figure unless you know the figure has a right angle.

If a coordinate proof requires calculations with fractions, choose coordinates that make the calculations simpler.

For example, use multiples of 2 when you are to find coordinates of a midpoint. Once you have assigned the coordinates of the vertices, the procedure for the proof is the same, except that your calculations will involve variables.

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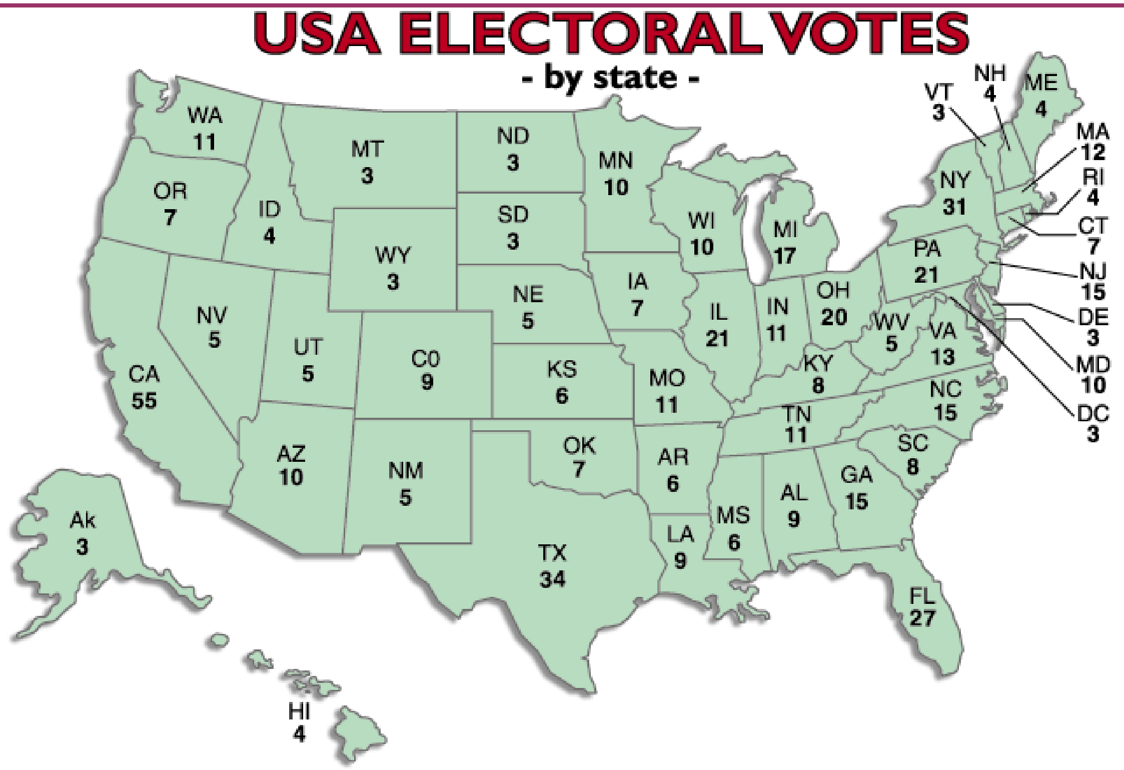
Remember!

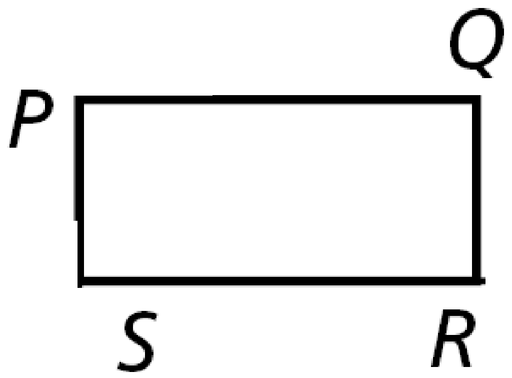
Because the *x*- and *y*-axes intersect at right angles, they can be used to form the sides of a right triangle.



Example 4: Writing a Coordinate Proof

Given: Rectangle *PQRS*

Prove: The diagonals are ≅.



Step 1 Assign coordinates to each vertex.

The coordinates of *P* are (0, *b*),

the coordinates of *Q* are (*a*, *b*),

the coordinates of *R* are (*a*, 0),

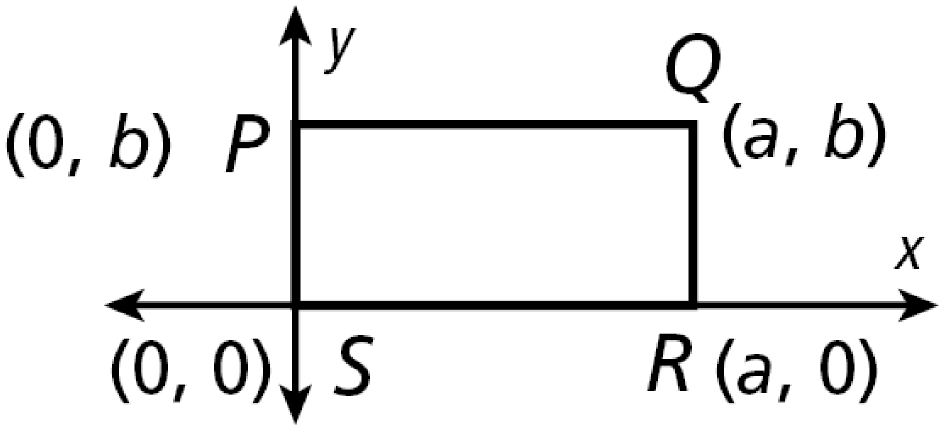
and the coordinates of *S* are (0, 0).

Step 2 Position the figure in the coordinate plane.

Example 4 Continued

Given: Rectangle *PQRS*

Prove: The diagonals are ≅



Step 3 Write a coordinate proof.

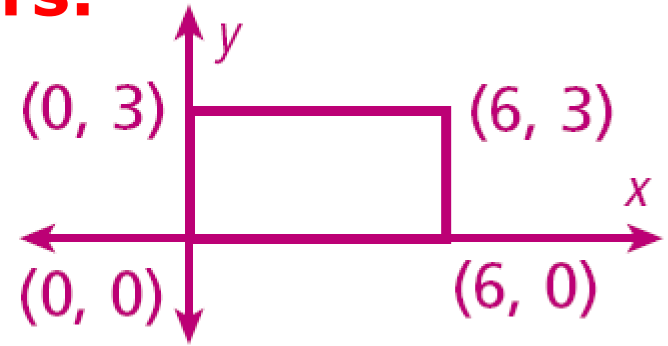
By the distance formula, $PR = \sqrt{a^2 + b^2}$, and $QS = \sqrt{a^2 + b^2}$. Thus the diagonals are ≅.

Lesson Quiz: Part I

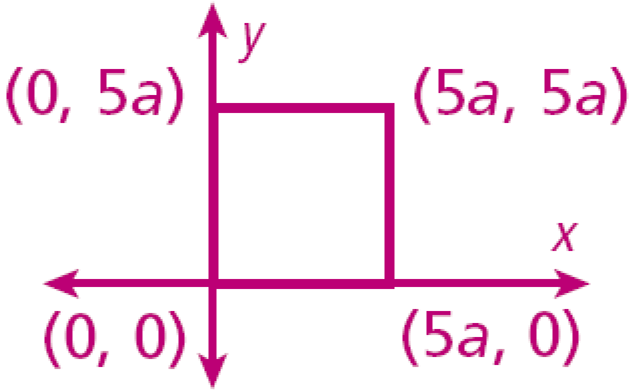
Position each figure in the coordinate plane.

Possible answers:

1. rectangle with a length of 6 units and a width of 3 units



2. square with side lengths of 5*a* units



Lesson Quiz: Part II

3. Given: Rectangle *ABCD* with coordinates *A*(0, 0), *B*(0, 8), *C*(5, 8), and *D*(5, 0). *E* is mdpt. of $\overline{BC}$, and *F* is mdpt. of $\overline{AD}$.

Prove: *EF* = *AB*

By the Midpoint Formula, the coordinates of *E* are $\left(\frac{5}{2}, 8\right)$.

and *F* are $\left(\frac{5}{2}, 0\right)$. Then *EF* = 8, and *AB* = 8.

Thus *EF* = *AB*.