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Geometric Transformations:

Translation: *slide*

Reflection: *mirror*

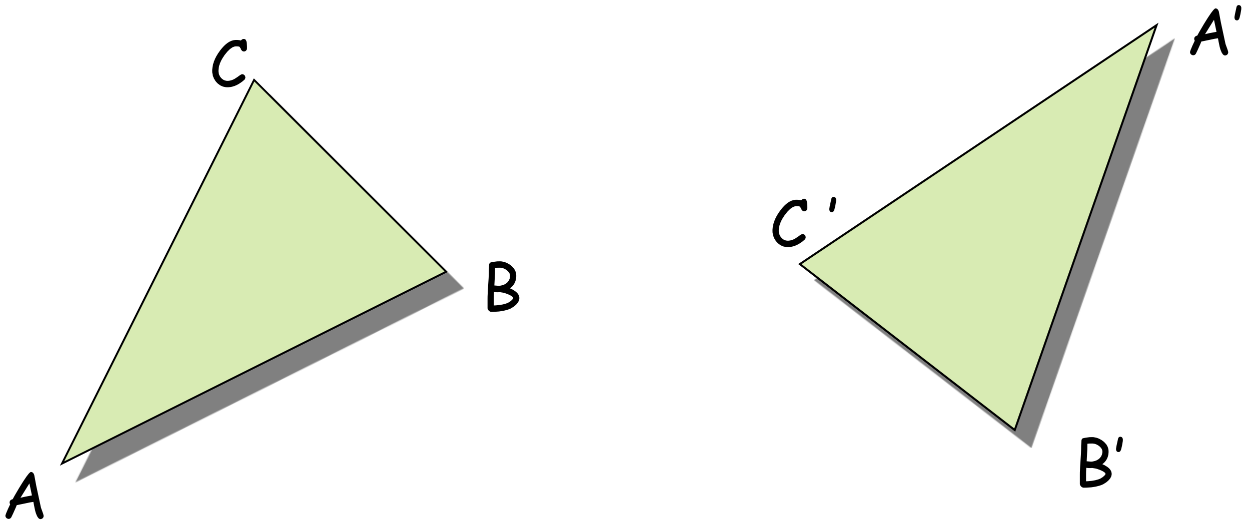
Rotation: *turn*

Dialation: *enlarge or reduce*

Notation:

Pre-Image: original figure

Image: after transformation.
Use prime notation



Isometry

AKA: *congruence transformation*

a transformation in which an original figure and its image are congruent.

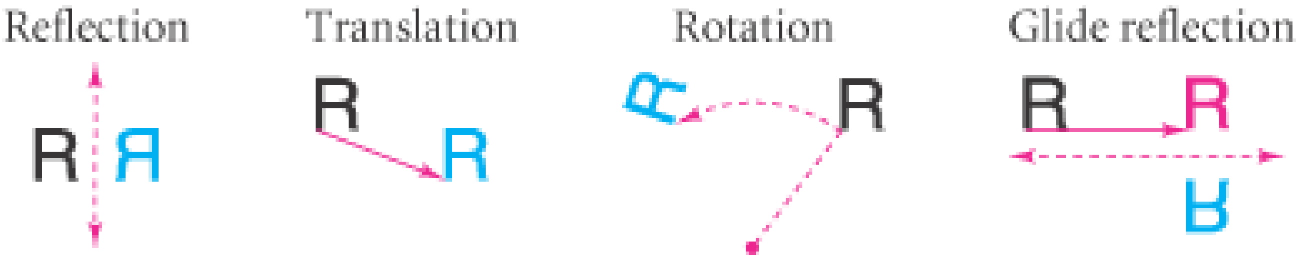
Theorems about isometries

FUNDAMENTAL THEOREM OF ISOMETRIES

Any any two congruent figures in a plane can be mapped onto one another by at most 3 reflections

ISOMETRY CLASSIFICATION THEOREM

There are only 4 isometries. They are:



TRANSLATION:

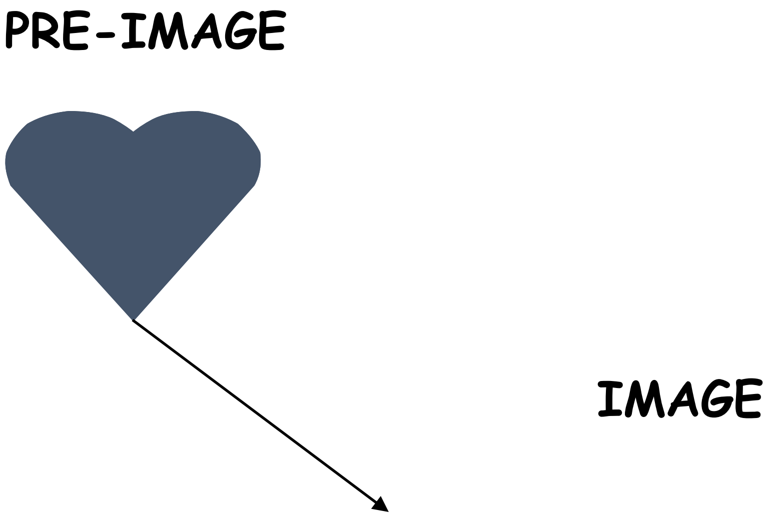
moves all points in a *plane*

a given direction

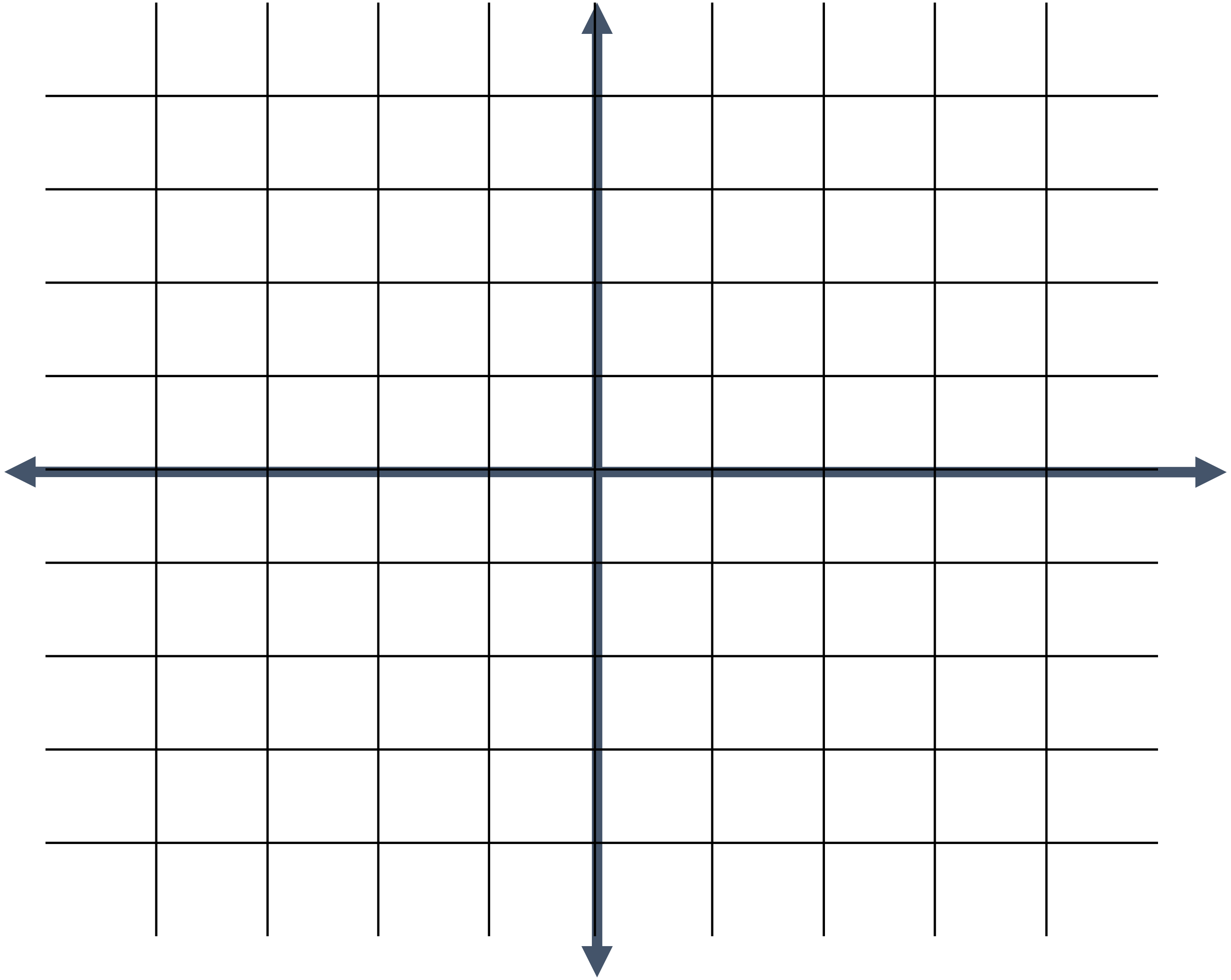
a fixed distance

TRANSLATION **VECTOR**:

Direction
Magnitude

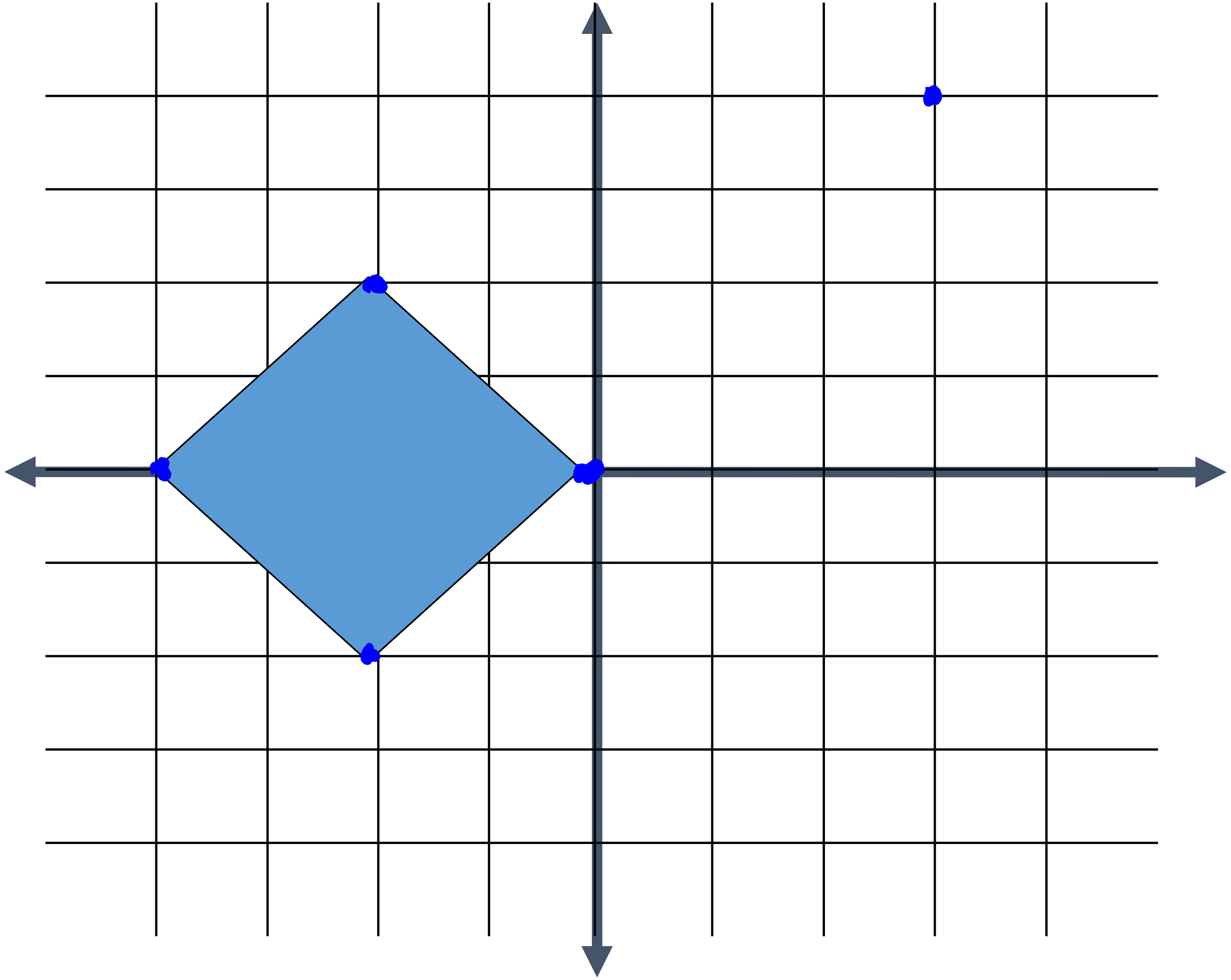


Translate by the **vector** <x, y>



x moves horizontal
y moves vertical

Translate by <3, 4>



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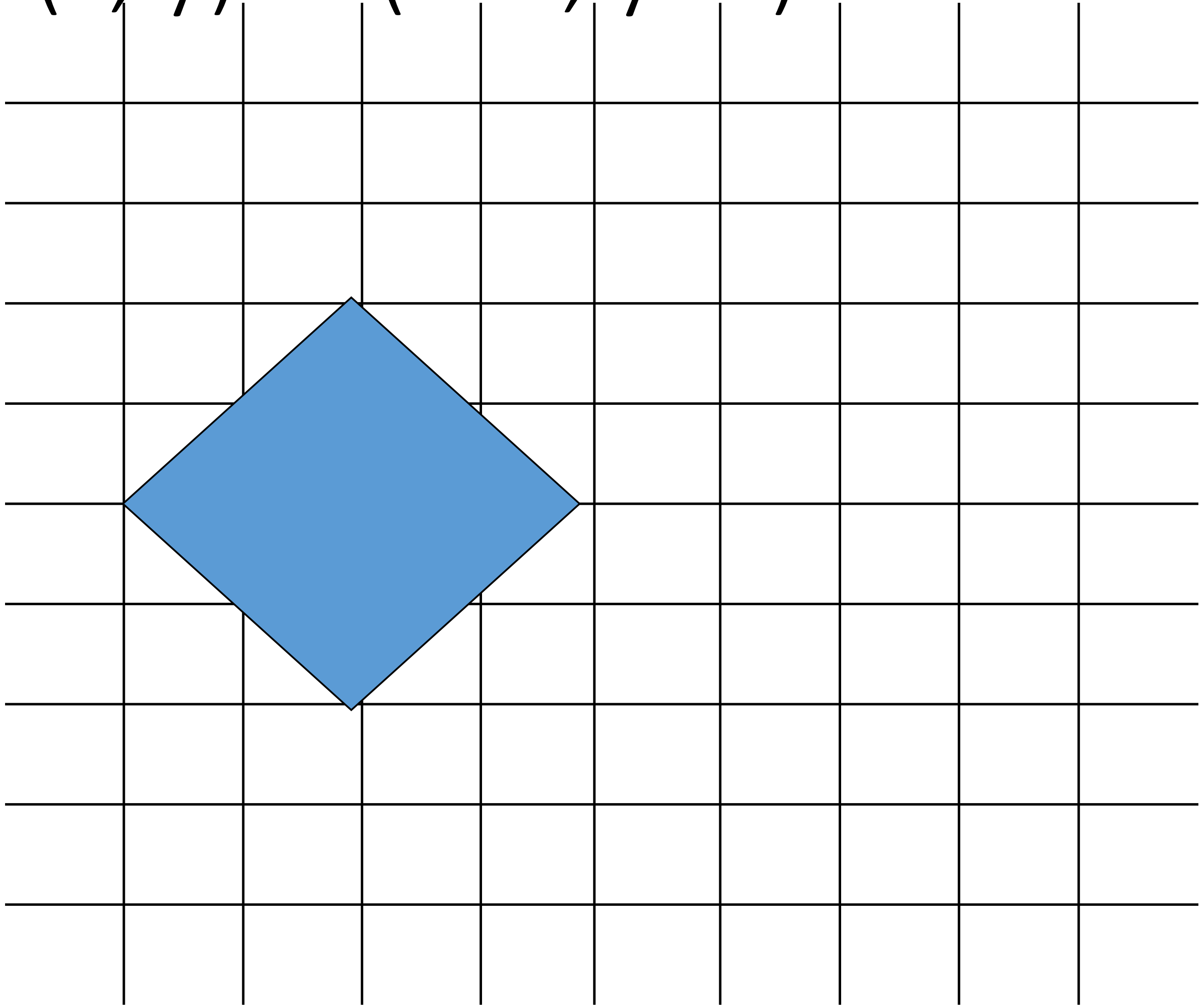
Different notation

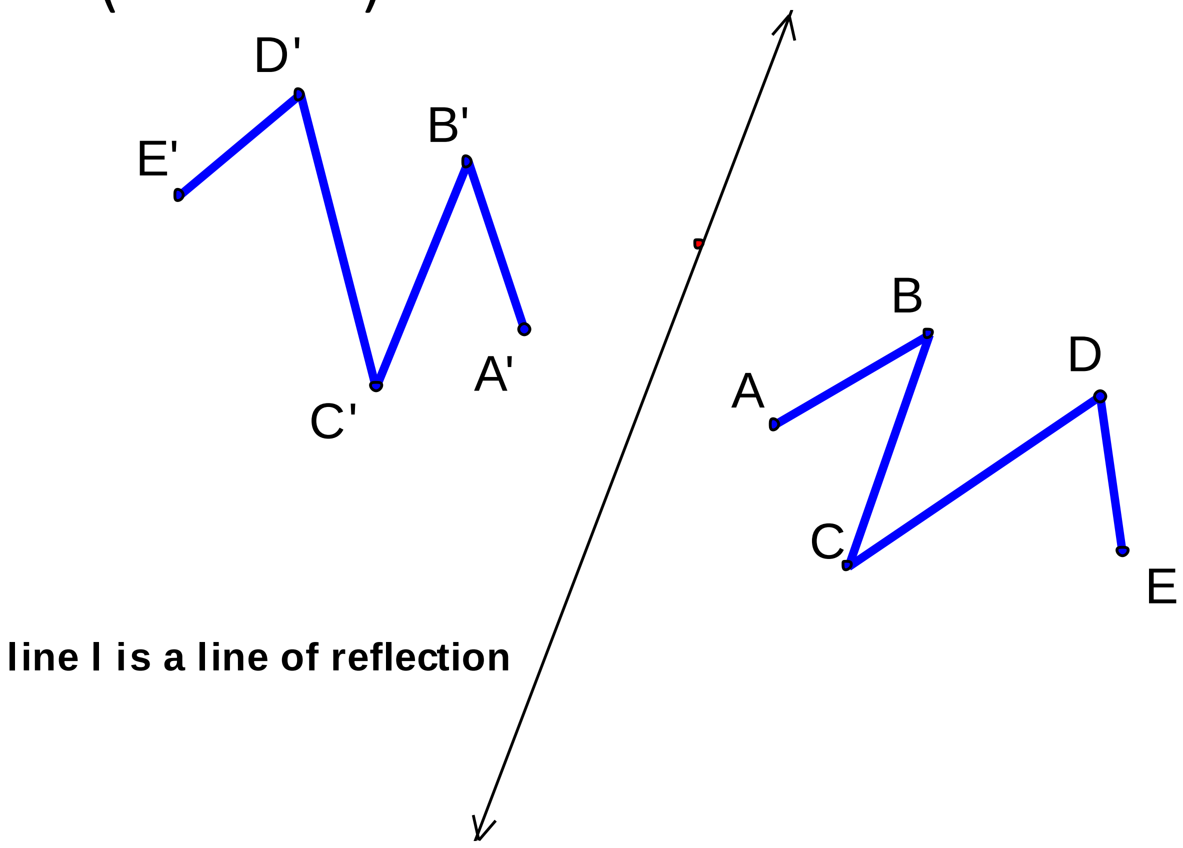
$T(x, y) \rightarrow (x+3, y+4)$



Translations ***PRESERVE***:

Size
Shape
Orientation

Reflection
over a line (mirror)



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Properties of reflections

PRESERVE

- Size (area, length, perimeter…)
- Shape

CHANGE

orientation (flipped)

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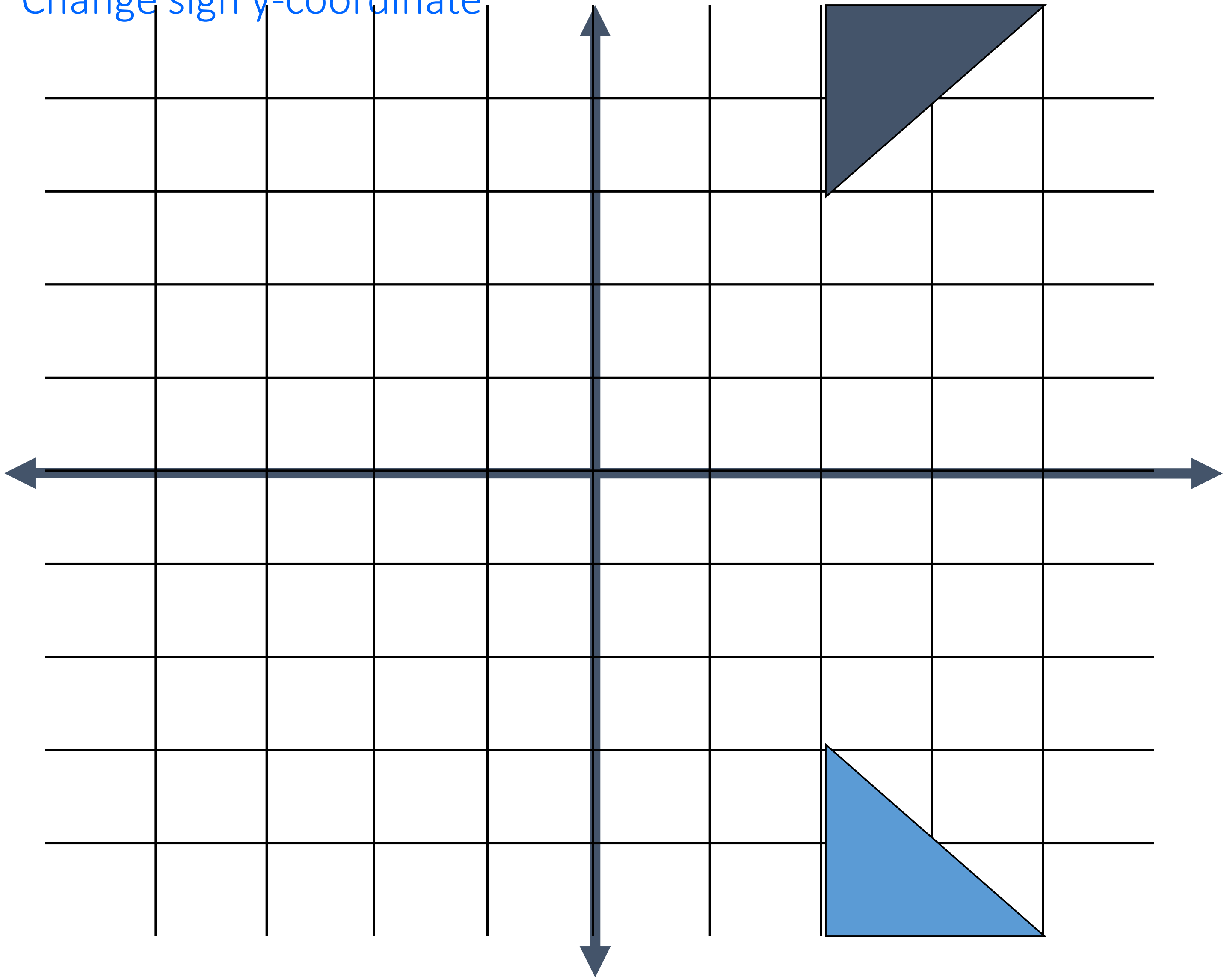
Reflect x-axis: (a, b) -> (a,-b)

Change sign y-coordinate



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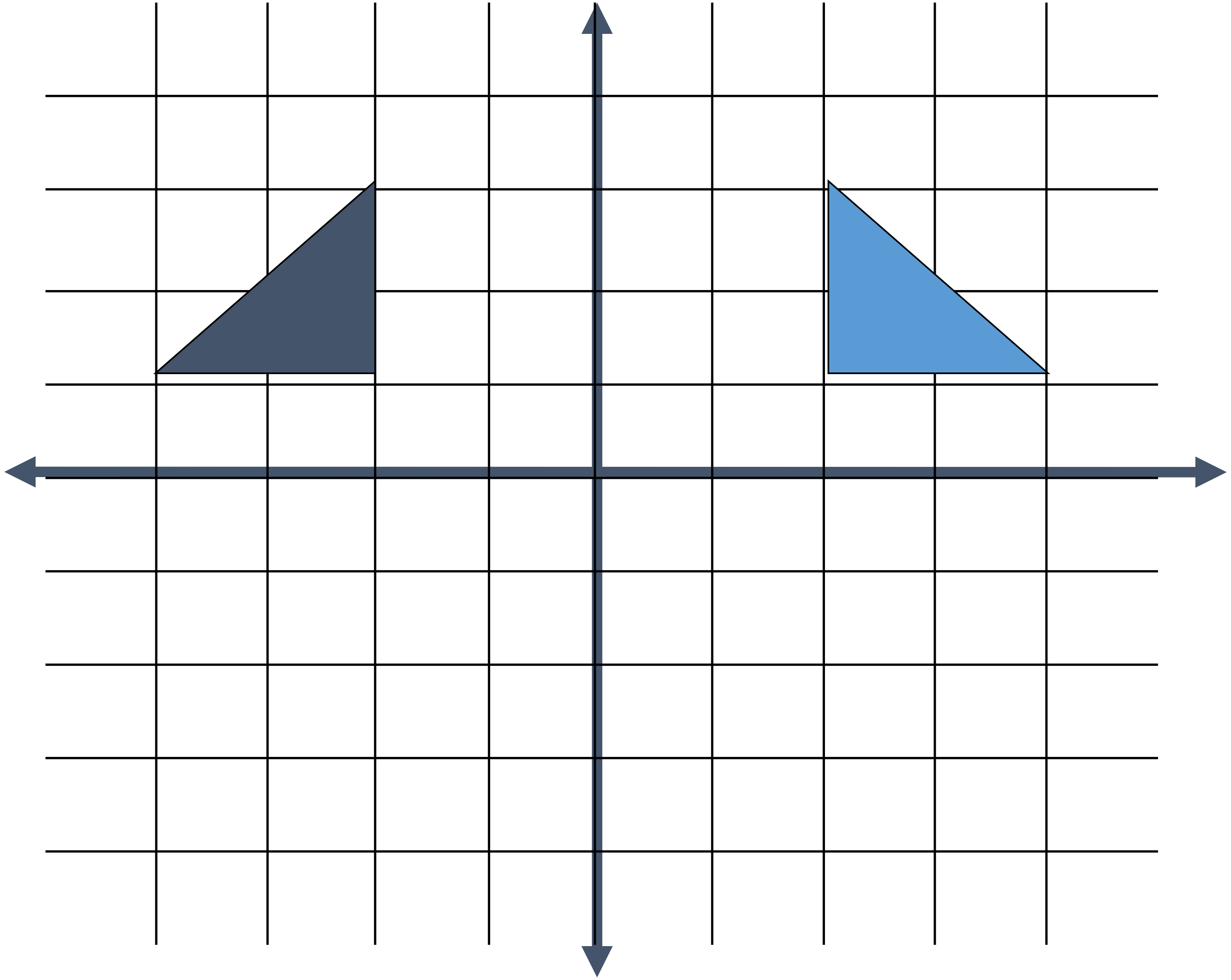
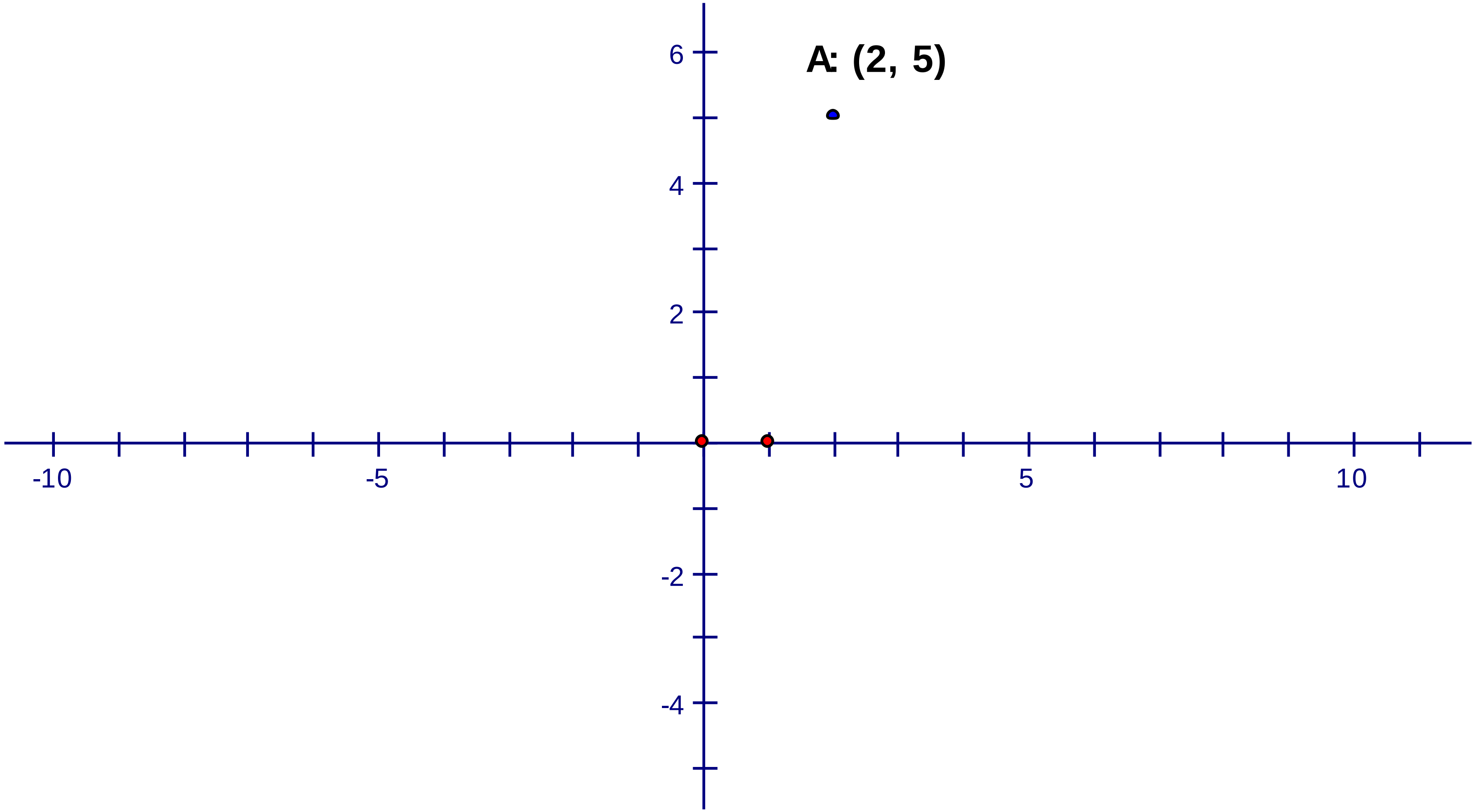
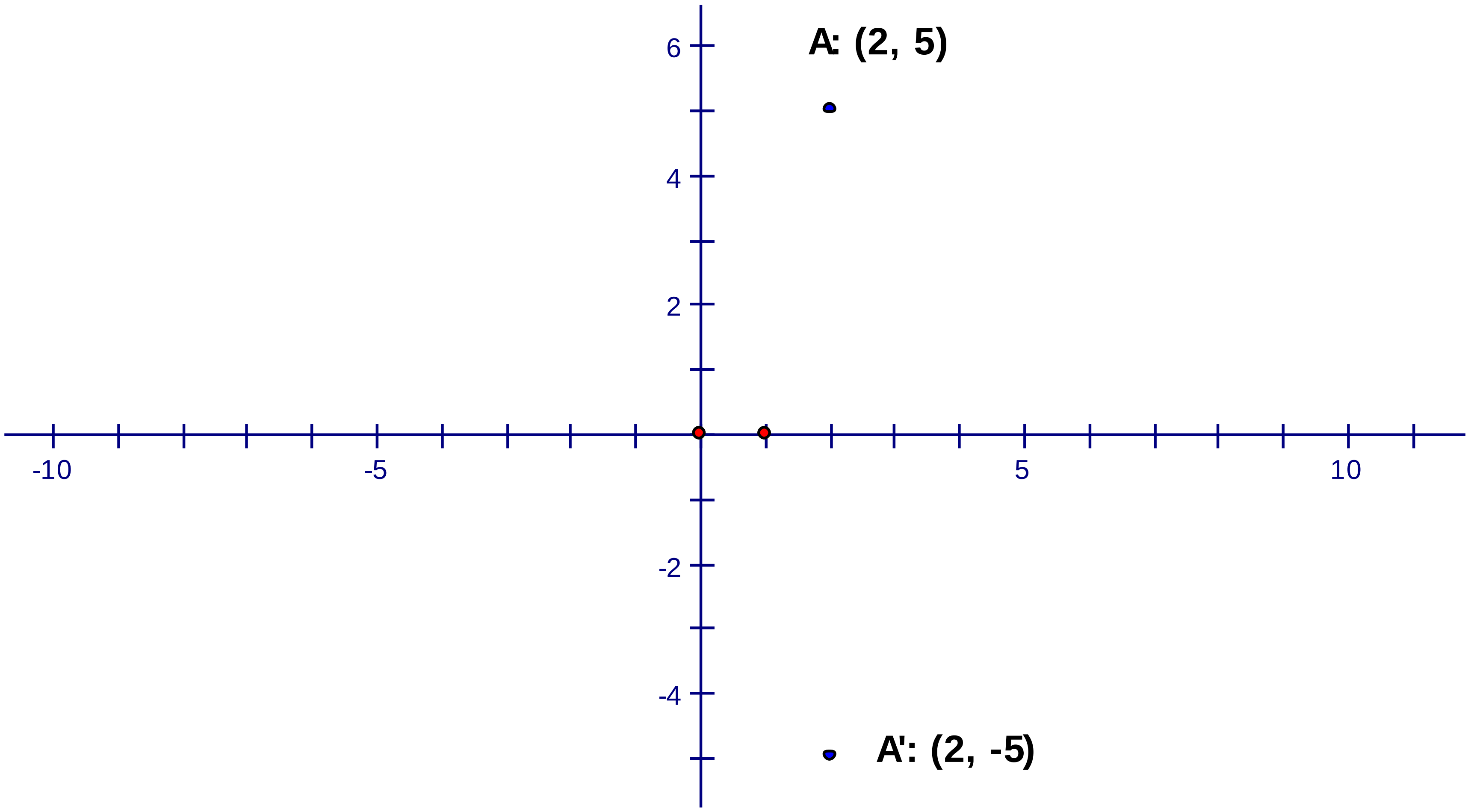
Reflect y-axis: **(a, b) -> (-a, b)**

Change sign on x coordinate




A: (2, 5)
6
4
2
-2
-4
-10
-5
5
10

X-axis reflection



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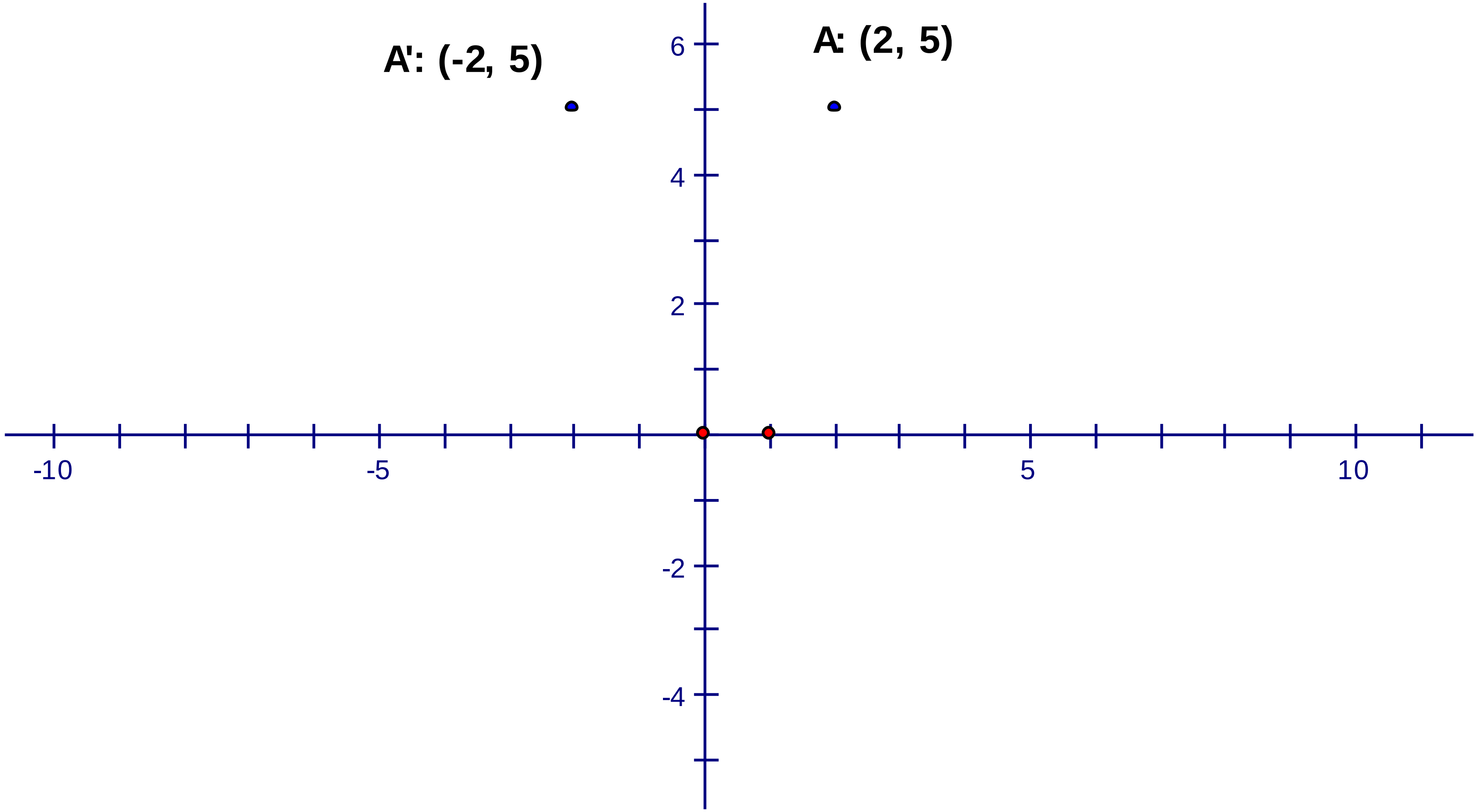
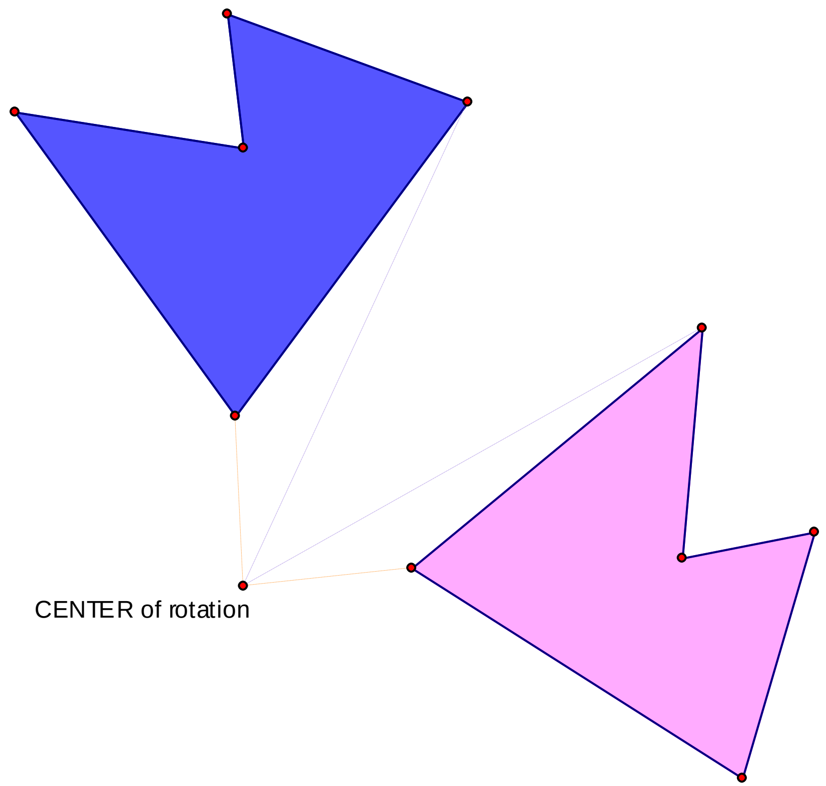
Y-axis reflection



Rotations have:

Center of rotation

Angle of rotation:



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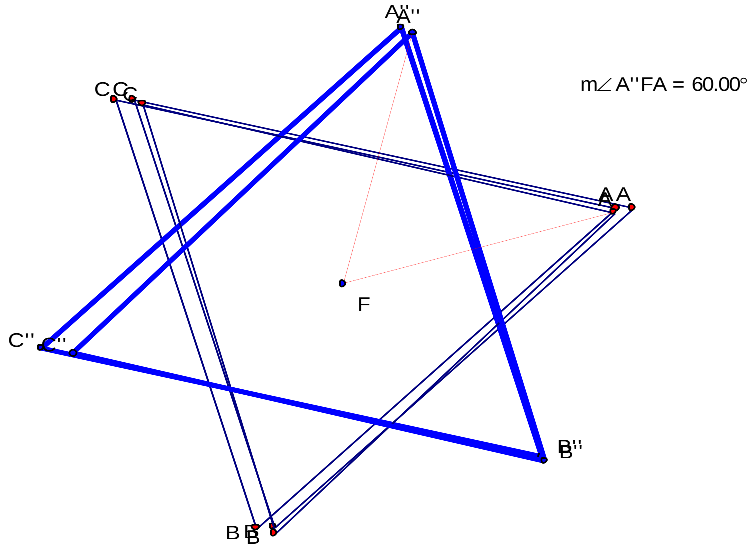
Example:
Rotate Triangle ABC
60 degrees clockwise about "its center"



- Find the image of A after a 120 degree rotation
- Find the image of A after a 180 degree rotation
- Find the image of A after a 240 degree rotation
- Find the image of A after a 300 degree rotation
- Find the image of A after a 360 degree rotation

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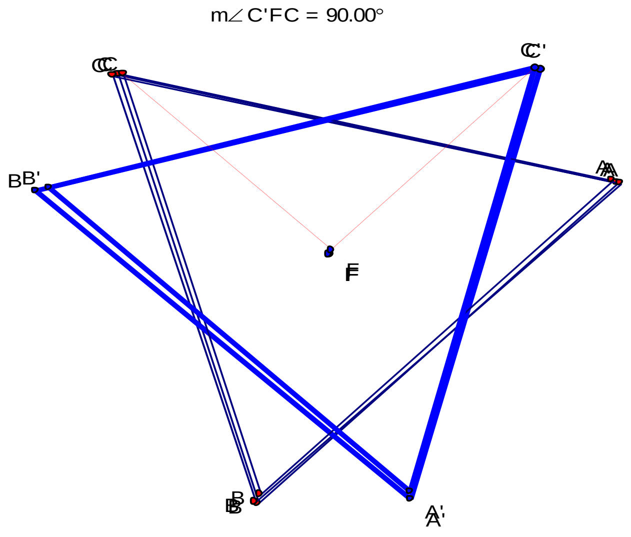
Rotated 90 degrees counterclockwise



ROTATIONS PRESERVE

SIZE

- Length of sides
- Measure of angles
- Area
- Perimeter

SHAPE

ORIENTATION

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Rotations on a coordinate plane about the origin

90°	(a, b) ->	(-b, a)
180°	(a, b) ->	(-a, -b)
270°	(a, b) ->	(b, -a)
360°	(a, b) ->	(a, b)

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Coordinate Geometry rules

Reflections

x axis	(a, b)	->	(a, -b)
y axis	(a, b)	->	(-a, b)
y=x	(a, b)	->	(b, a)

Rotations about the origin

90°	(a, b)	->	(-b, a)
180°	(a, b)	->	(-a, -b)
270°	(a, b)	->	(b, -a)
360°	(a, b)	->	(a, b)

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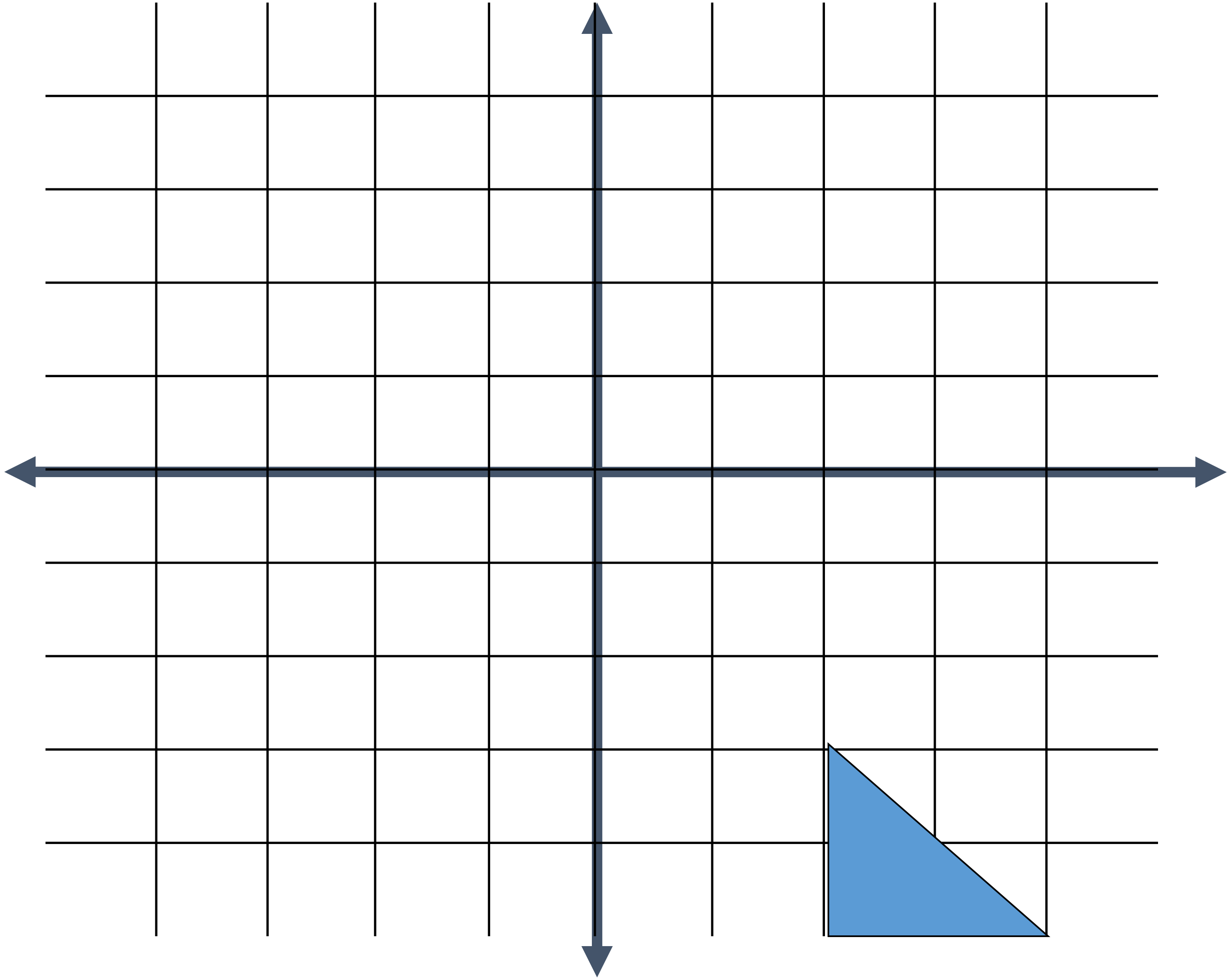
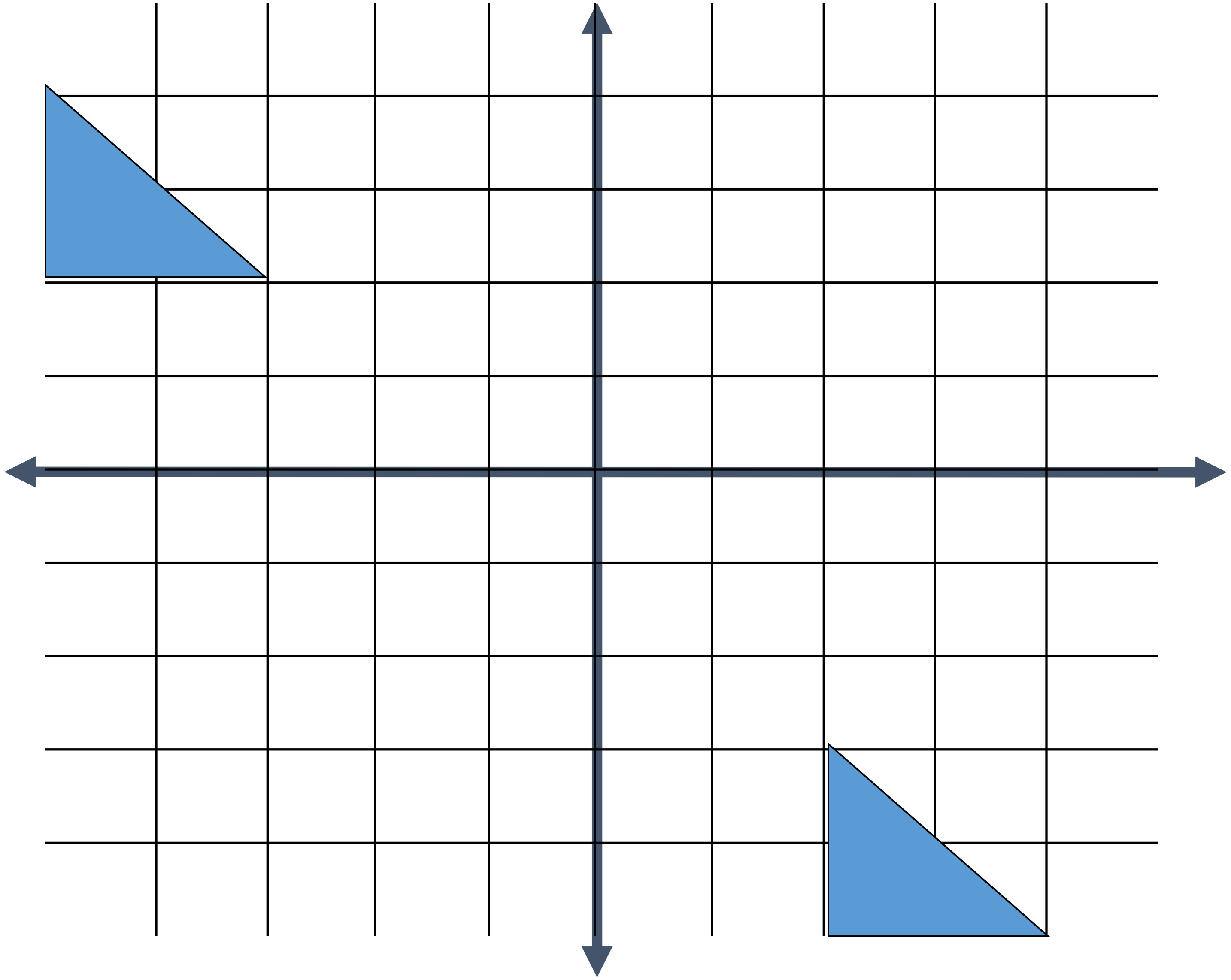
GLIDE REFLECTIONS

You can combine different Geometric Transformations…

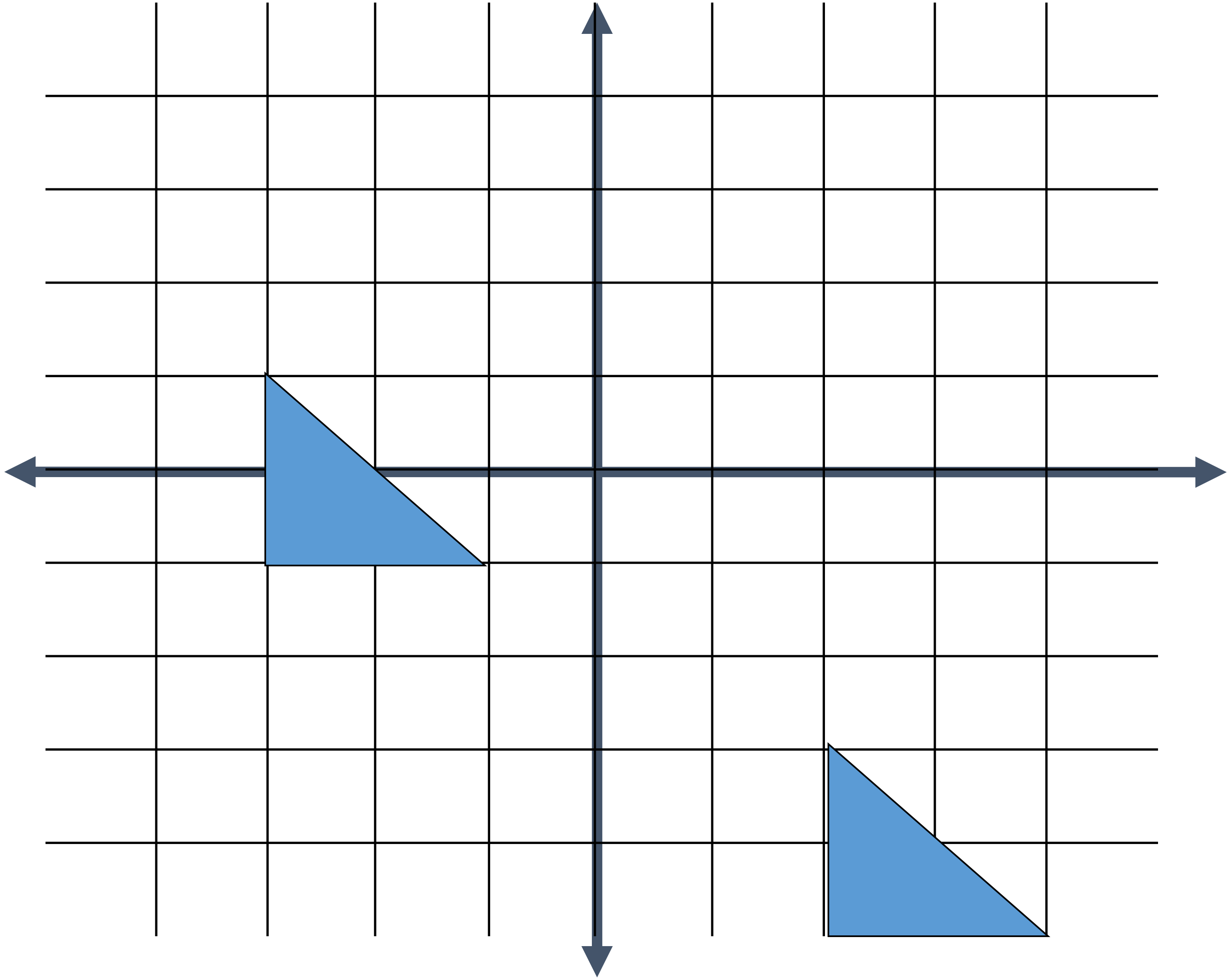
Practice: Reflect over y = x then translate by the vector <2, -3>



After Reflection…



After Reflection and translation…



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Symmetry

Line Symmetry

If a figure can be reflected onto itself over a line.

Rotational Symmetry

If a figure can be rotated about some point onto itself through a rotation between 0 and 360 degrees

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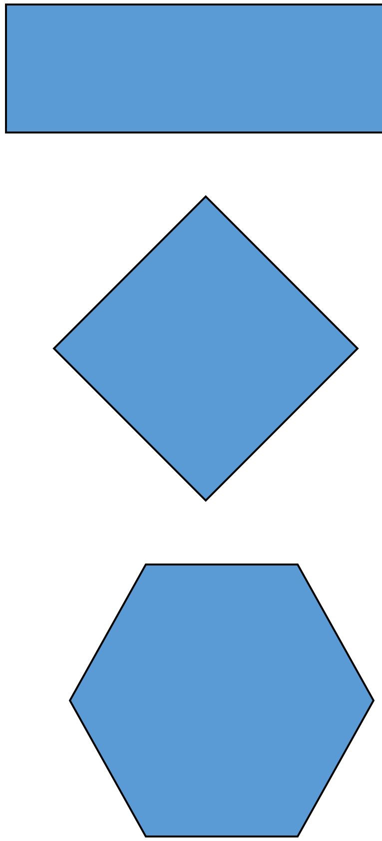
What kinds of symmetry do each of the following have?



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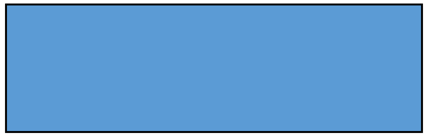
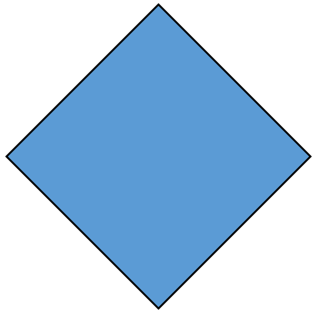
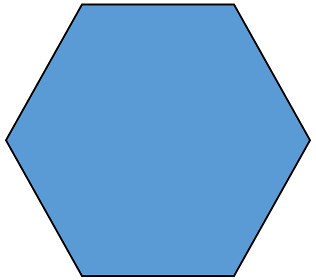
What kinds of symmetry do each of the following have?



Rotational (180) Point Symmetry



Rotational (90, 180, 270)
Point Symmetry



Rotational (60, 120, 180, 240, 300)
Point Symmetry

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INDIRECT PROOF

If ~q then ~p

1. Assume that the conclusion is **FALSE**.
2. Reason to a **contradiction**.

If n>6 then the regular polygon will not tessellate.

ASSUME: The polygon tessellates

SHOW: n *can not* be >6

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Indirect proof

Regular polygons with n>6 sides **will not tessellate**

Proof:

Assume a polygon with n>6 sides **<u>will tessellate</u>**.

This means that n*one interior <measure will equal 360

- IF n = 3 there are 6 angles about center point
- IF n = 4 there are 4 angles about center point
- IF n = 6 there are 3 angles about center point

•Therefore, if n>6 then there must be <u>fewer than 3</u> angles about the center point. In other words, there must be 2 or fewer. If there are 2 angles about the center point then each angle must measure 180 to sum to 360

•But no regular polygon exists whose interior angle measures 180 (int. < sum must be LESS than 180). Therefore, the polygon can not tessellate.