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Geometry

Glide Reflections and Compositions

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Goals

- Identify glide reflections in the plane.
- Represent transformations as compositions of simpler transformations.

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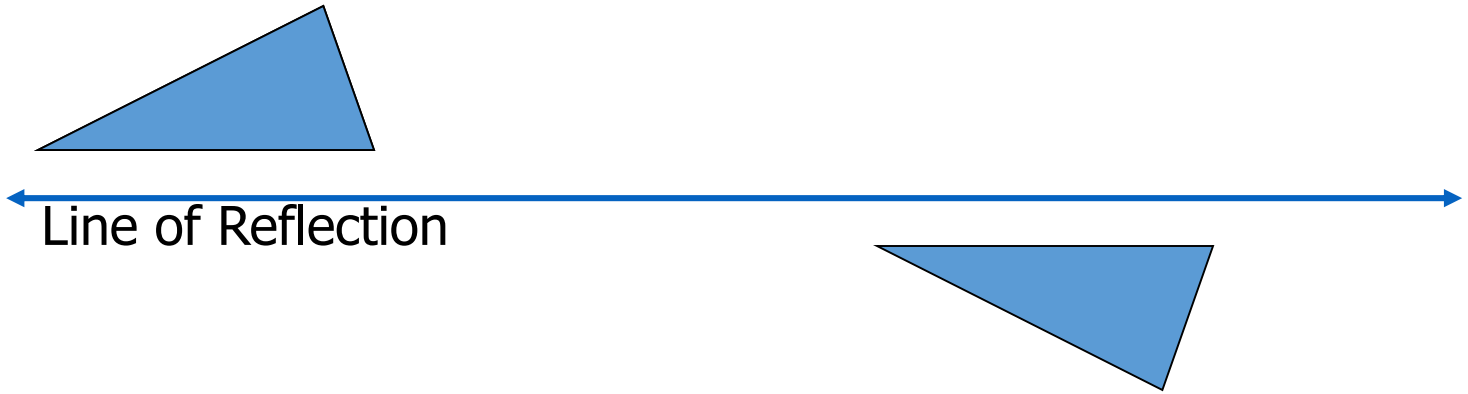
Glide Reflection

- A glide reflection is a transformation where a translation (the glide) is followed by a reflection.



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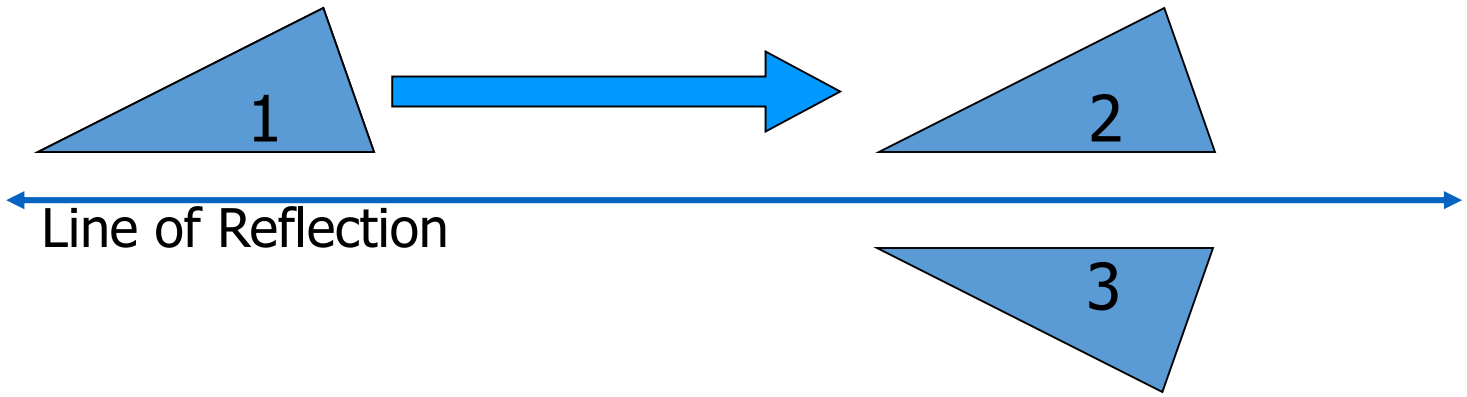
Glide Reflection

1. A translation maps P onto P’.
2. A reflection in a line k parallel to the direction of the translation maps P’ to P’’.



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Example

Find the image of $\triangle ABC$ after a glide reflection.

A(-4, 2), B(-2, 5), C(1, 3)

Translation: $(x, y) \rightarrow (x + 7, y)$

Reflection: in the x-axis

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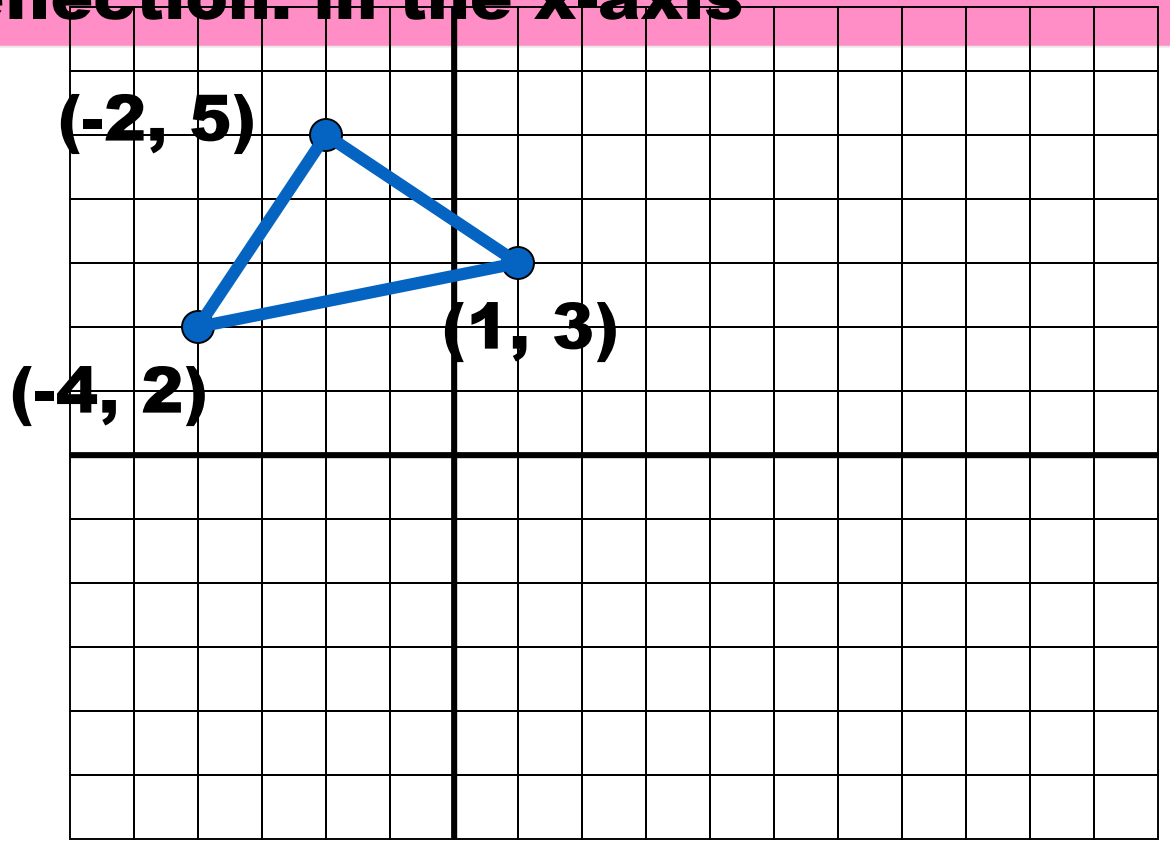

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Find the image of △ABC after a glide reflection.

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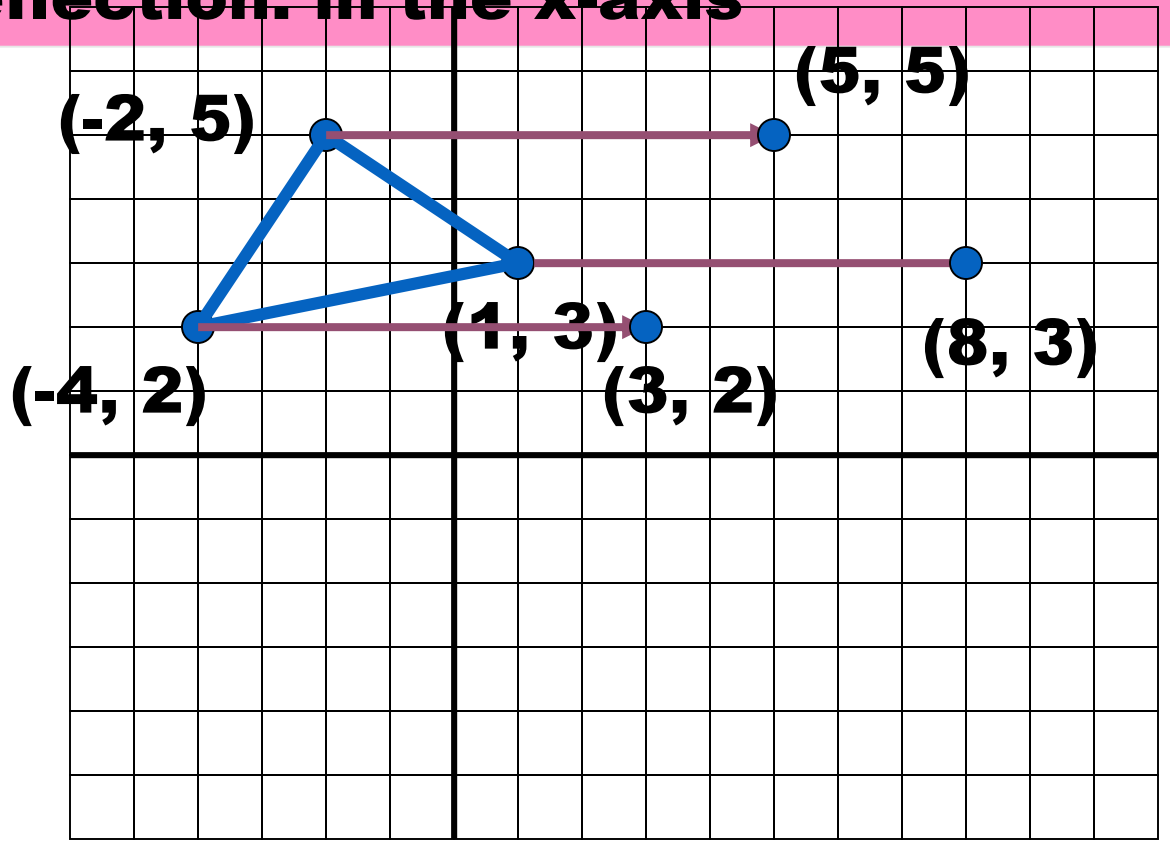

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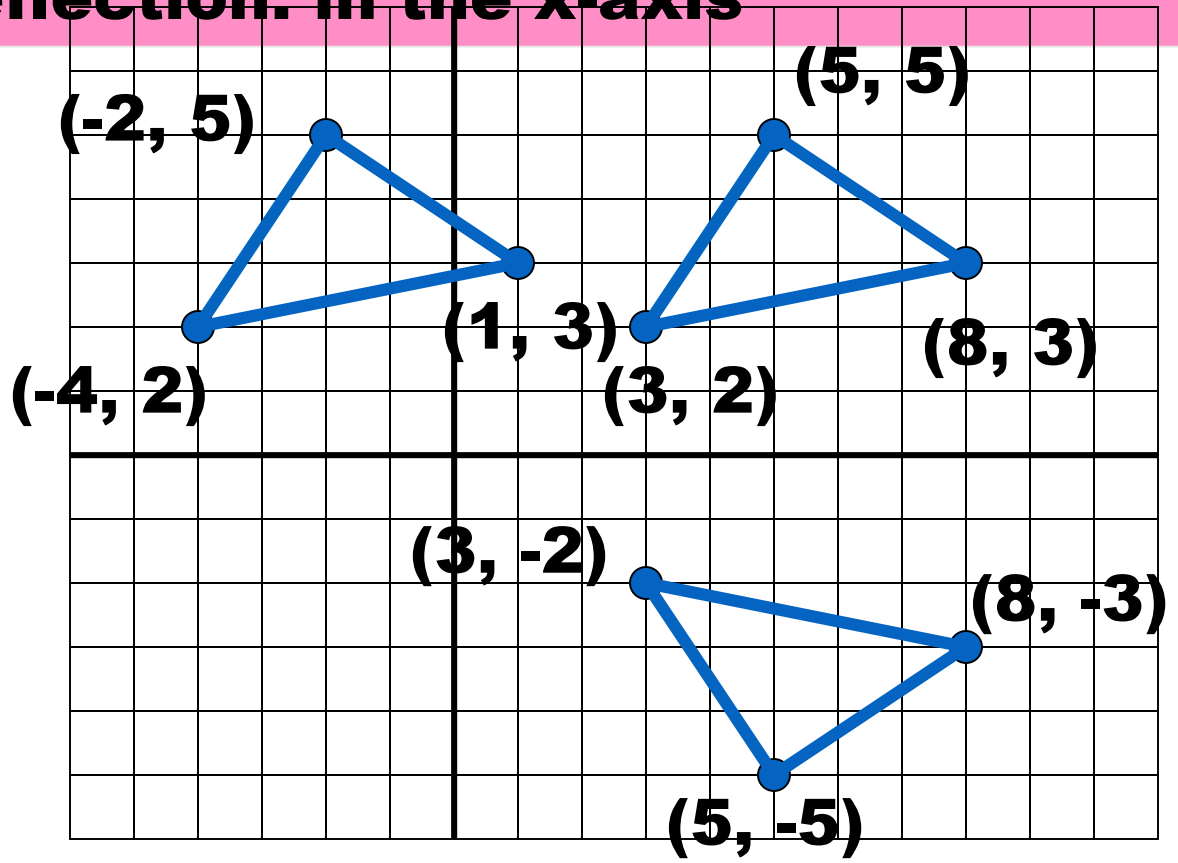

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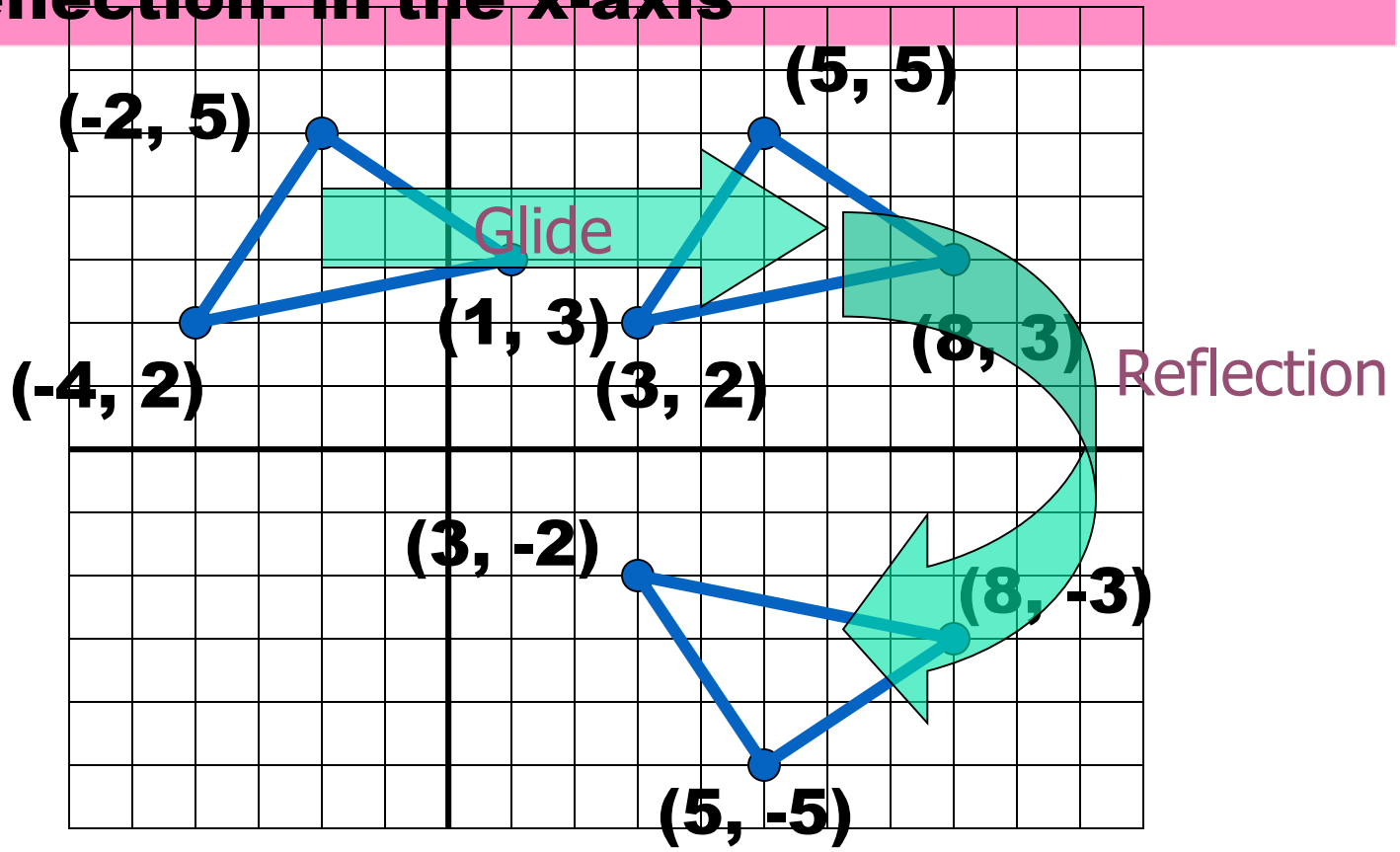

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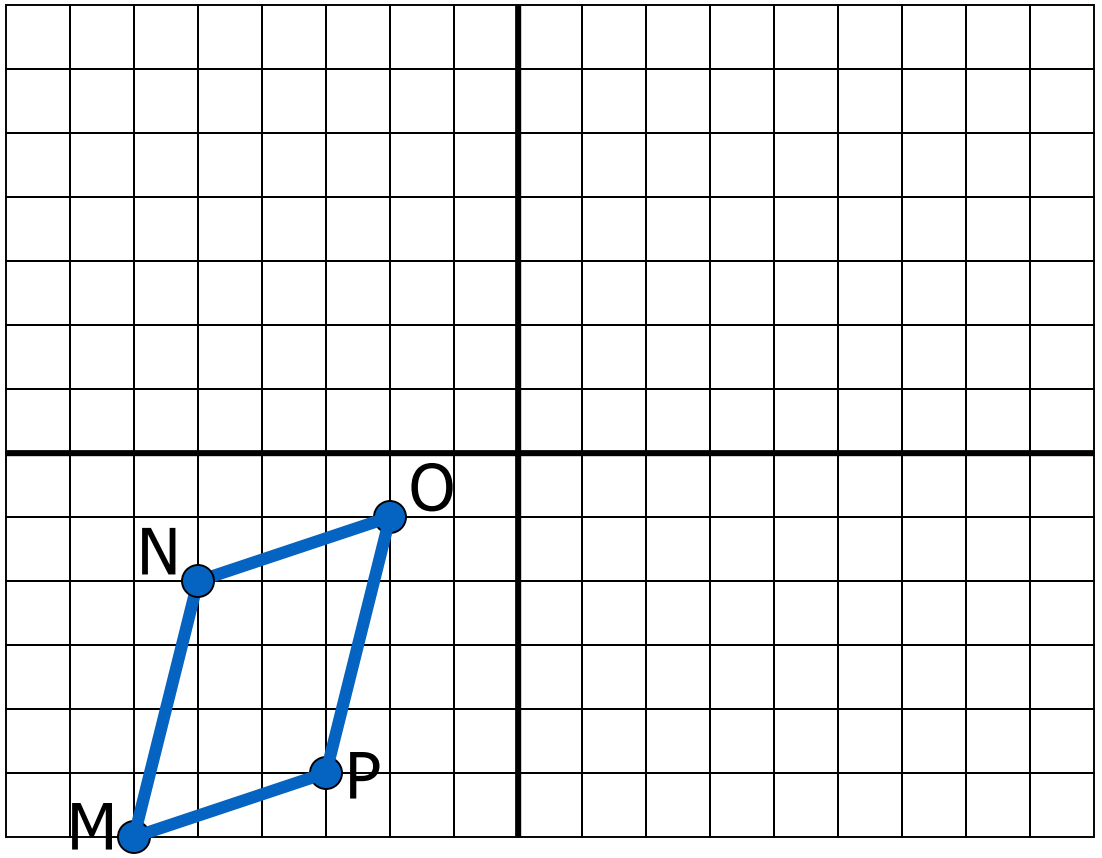
You do it.

- Locate these four points:
- M(-6, -6)
- N(-5, -2)
- O(-2, -1)
- P(-3, -5)
- Draw MNOP



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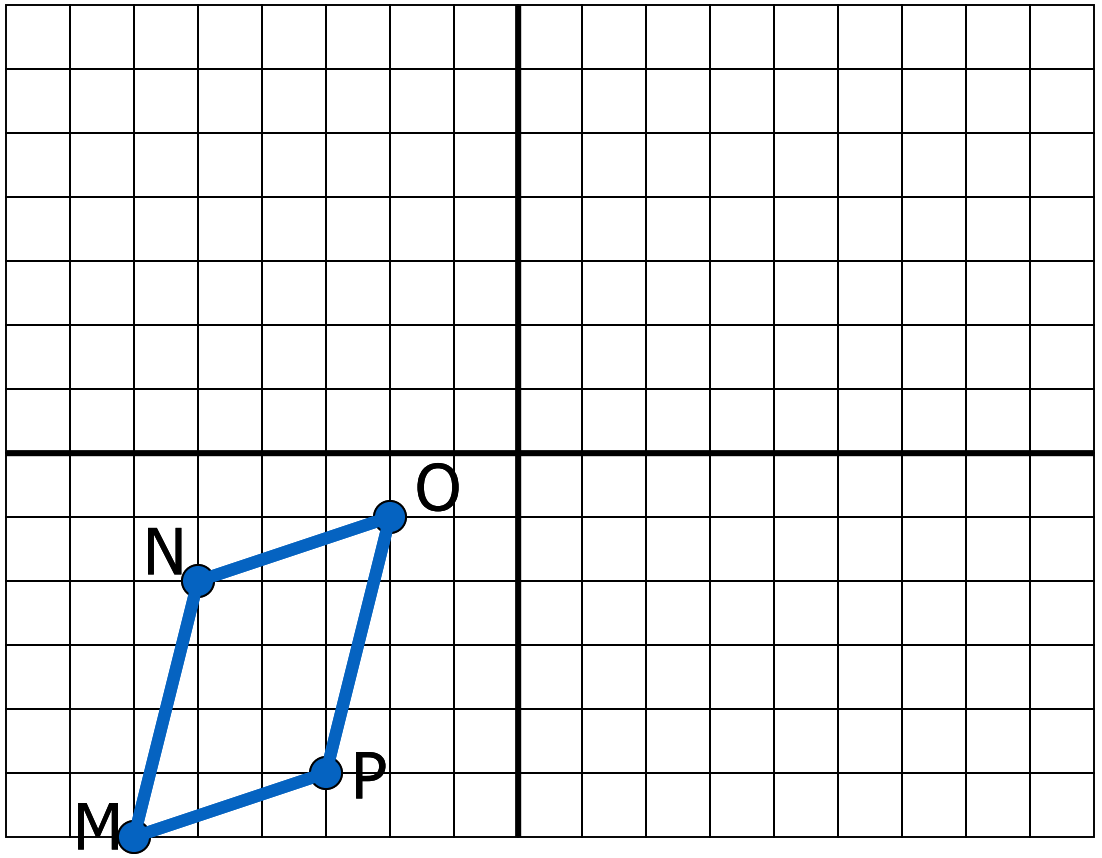
You do it.

- Translate by $\langle 0, 7 \rangle$.



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You do it.

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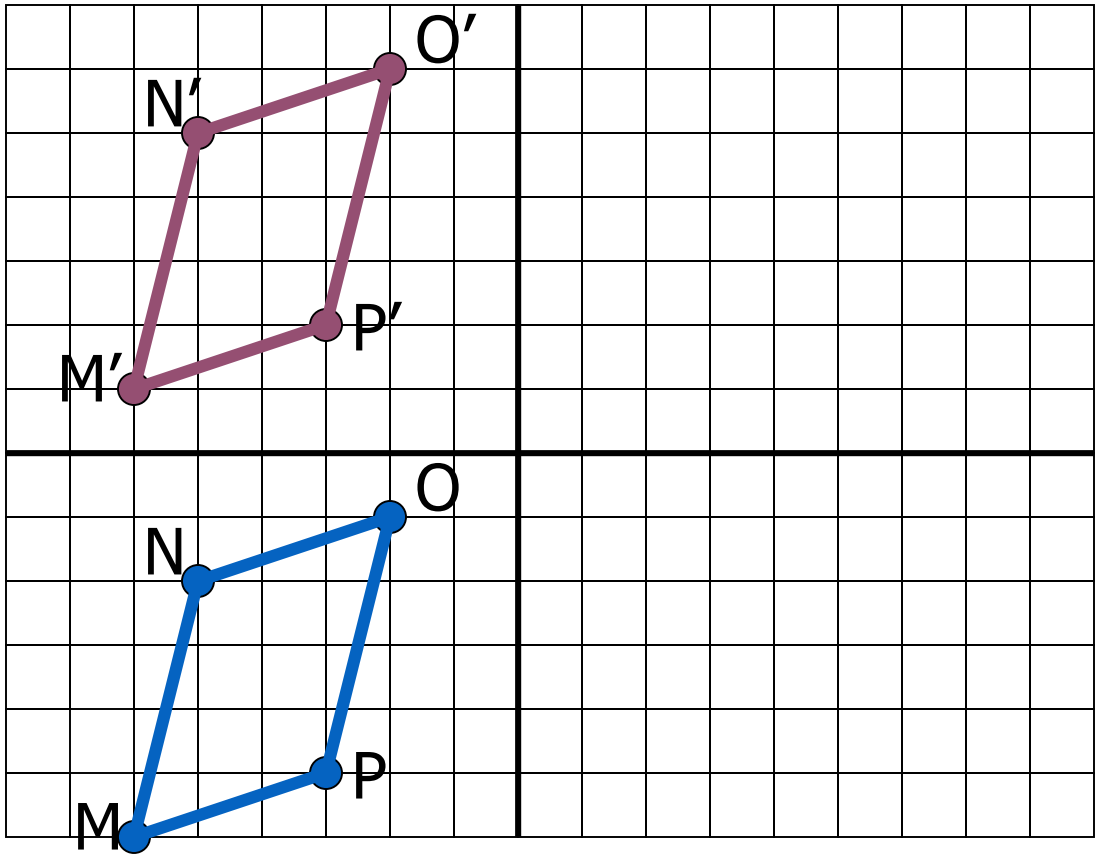

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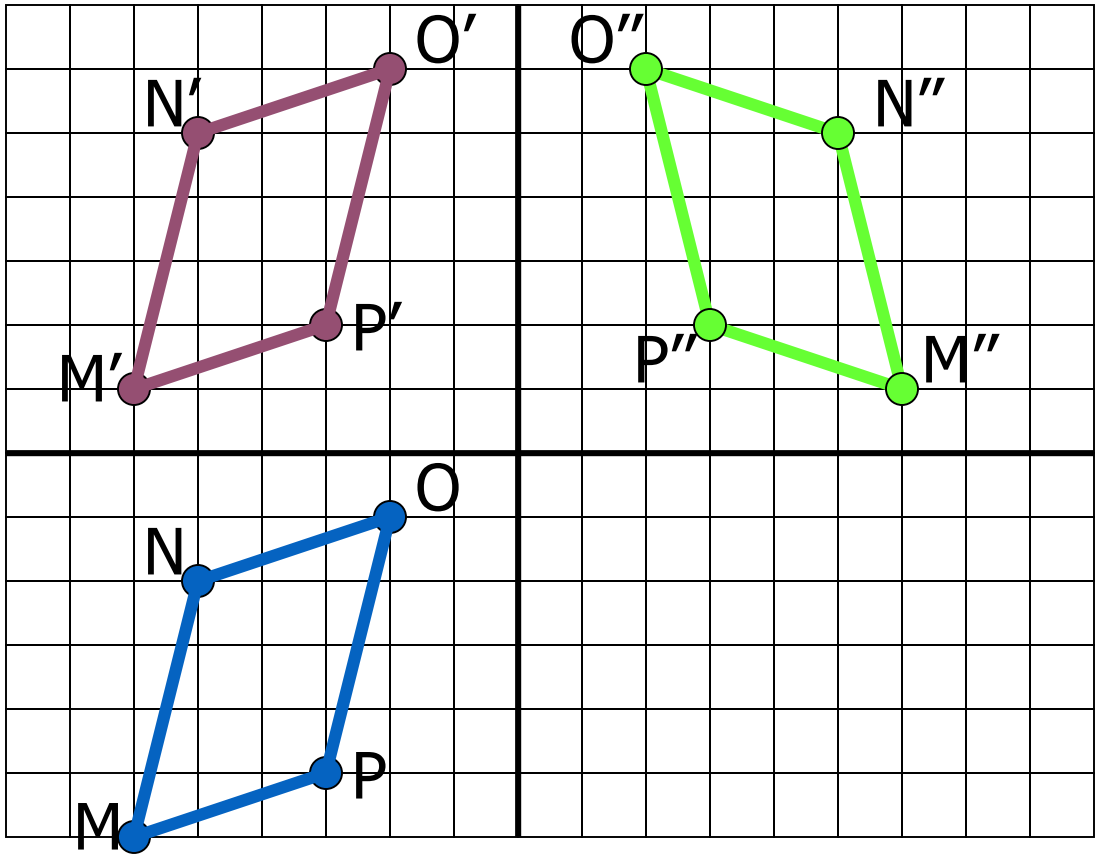
You do it.

- Reflect over y-axis.



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Compositions

- A composition is a transformation that consists of two or more transformations performed one after the other.

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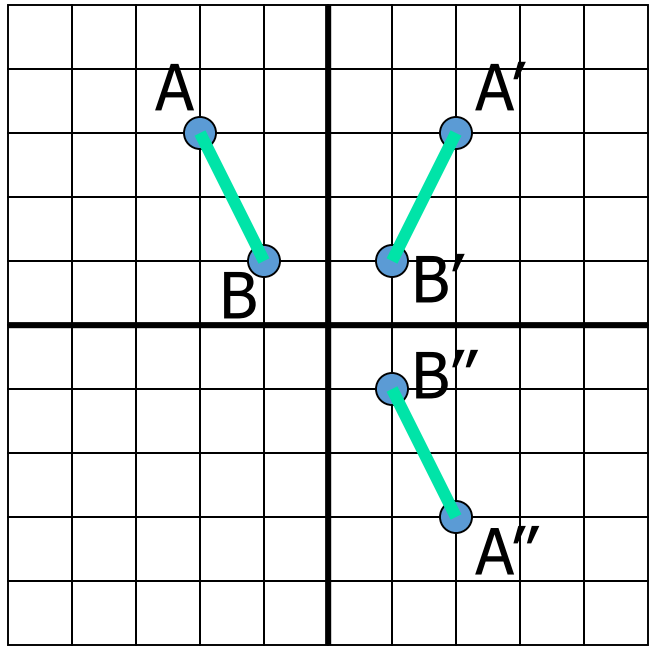
Composition Example

1. Reflect $\overline{AB}$ in the y-axis.
2. Reflect $\overline{A'B'}$ in the x-axis.



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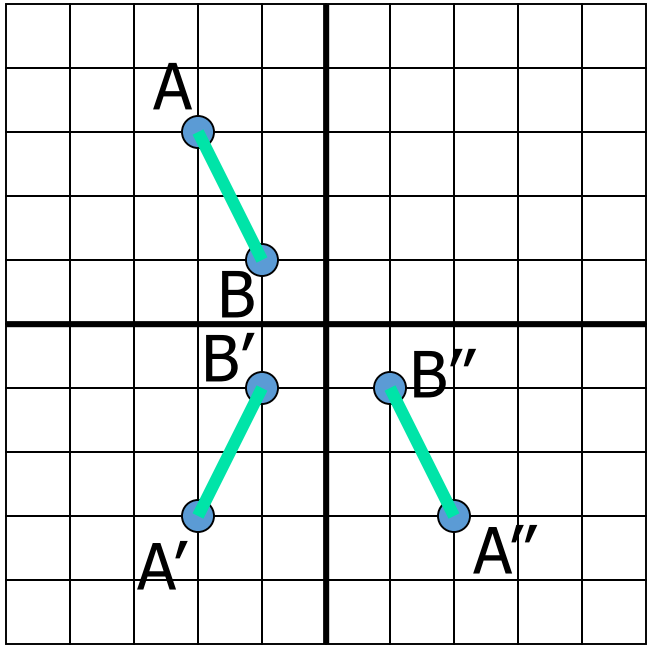
Try it in a different order.

1. Reflect $\overline{AB}$ in the x-axis.
2. Reflect $\overline{A'B'}$ in the y-axis.



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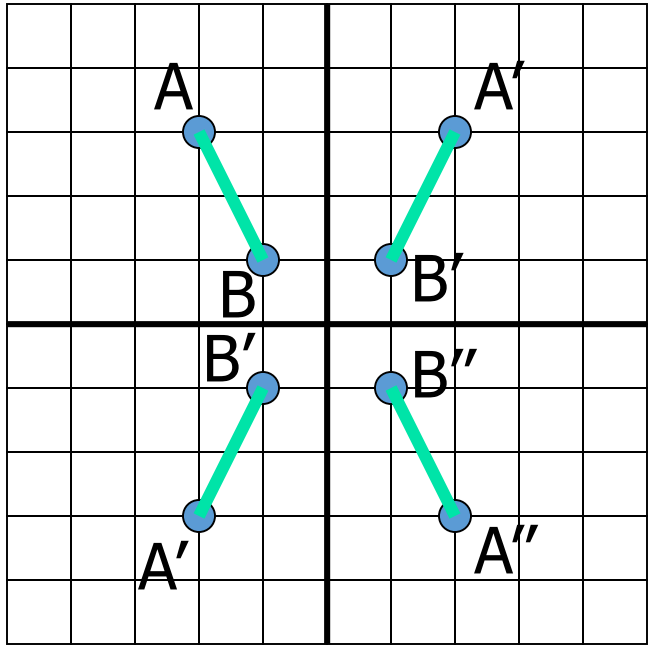
The order doesn't matter.



This composition is commutative.

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Commutative Property

- $a + b = b + a$
- $25 + 5 = 5 + 25$
- $ab = ba$
- $4 \times 25 = 25 \times 4$
- Reflect in y, reflect in x is equivalent to reflect in x, reflect in y.

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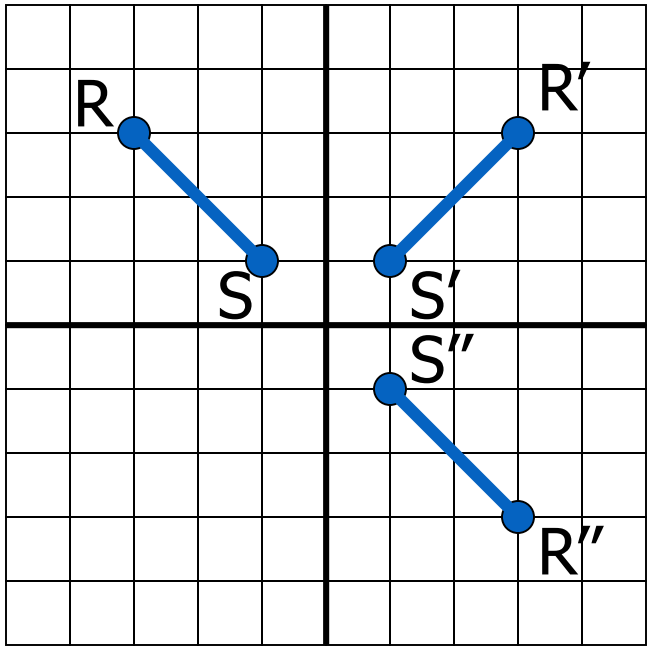
Are all compositions commutative?

Rotate $\overline{RS}$ 90° CW.

Reflect $\overline{R'S'}$ in x-axis.



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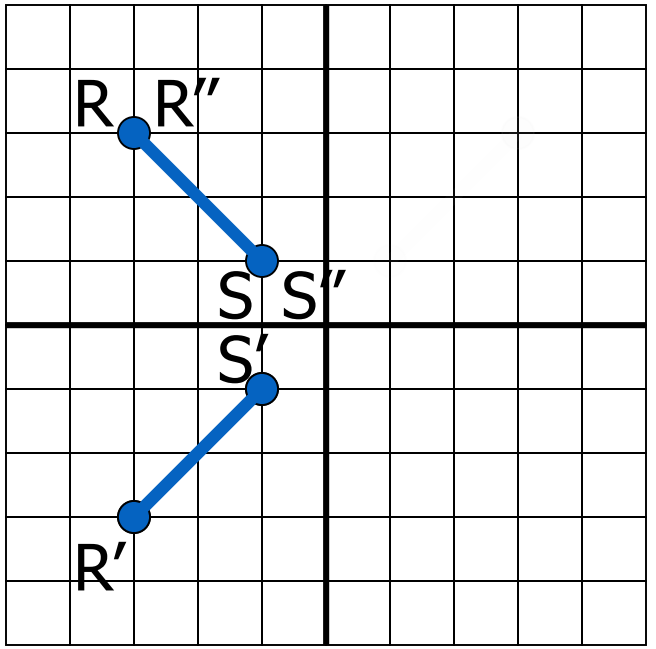
Reverse the order.

Reflect $\overline{RS}$ in the x-axis.

Rotate $\overline{R'S'}$ 90° CW.



All compositions are NOT commutative. Order matters!

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Compositions & Isometries

- If each transformation in a composition is an isometry, then the composition is an isometry.
- A Glide Reflection is an isometry.

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Example

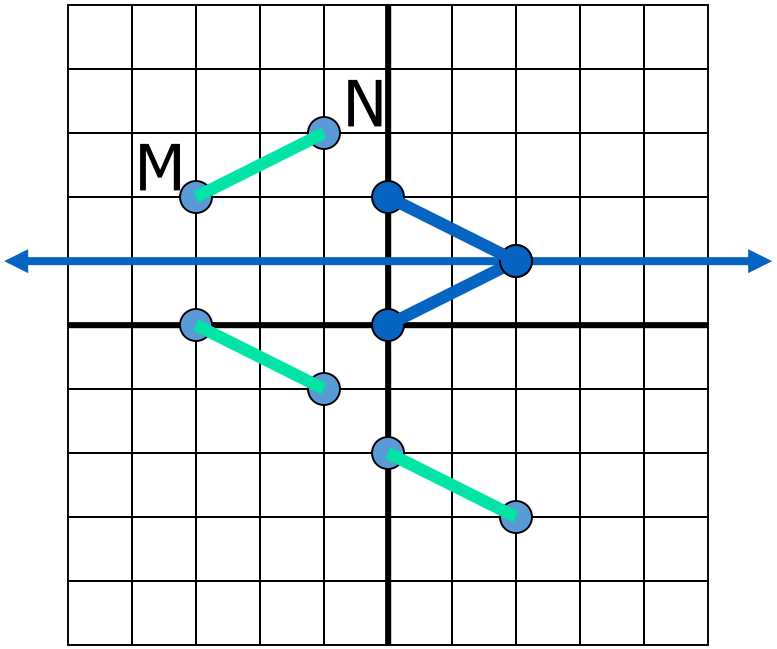
Reflect $\overline{MN}$ in the line y = 1.

Translate using vector ⟨3, -2⟩.

Now reverse the order:

Translate $\overline{MN}$ using ⟨3, -2⟩.

Reflect in the line y = 1.



Both compositions are isometries, but the composition is not commutative.

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Summary

- A Glide-Reflection is a composition of a translation followed by a reflection.
- Some compositions are commutative, but not all.