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Areas of Regular Polygons

Finding the area of an equilateral triangle

The area of any triangle with base length b and height h is given by

$$A = \frac{1}{2}bh$$

The following formula for equilateral triangles; however, uses ONLY the side length.

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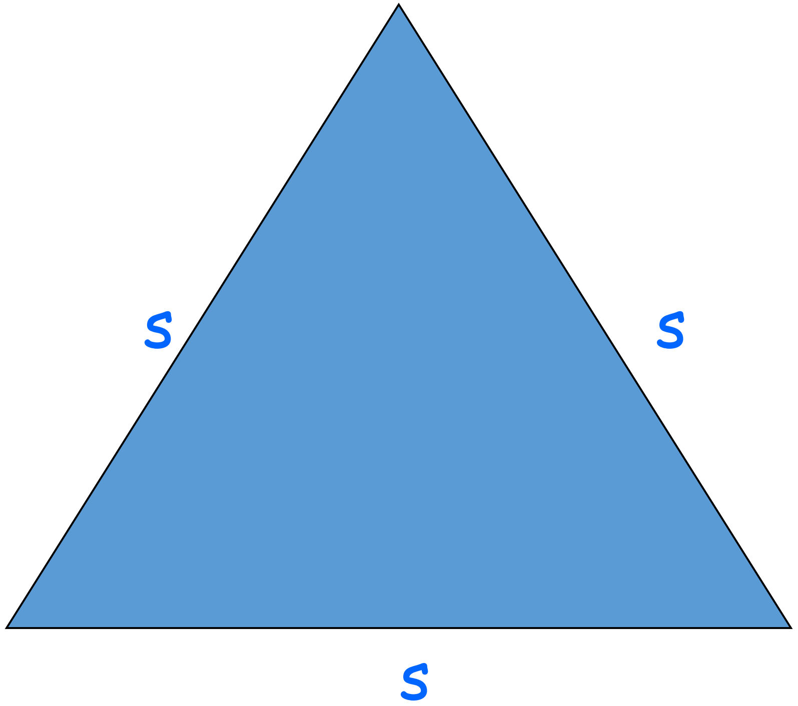
Area of an equilateral triangle

- The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$

$A = \frac{1}{4}\sqrt{3}\, s^2$



$A = \frac{1}{4}\sqrt{3}\, s^2$

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Finding the area of an Equilateral Triangle

- Find the area of an equilateral triangle with 8 inch sides.

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Finding the area of an Equilateral Triangle

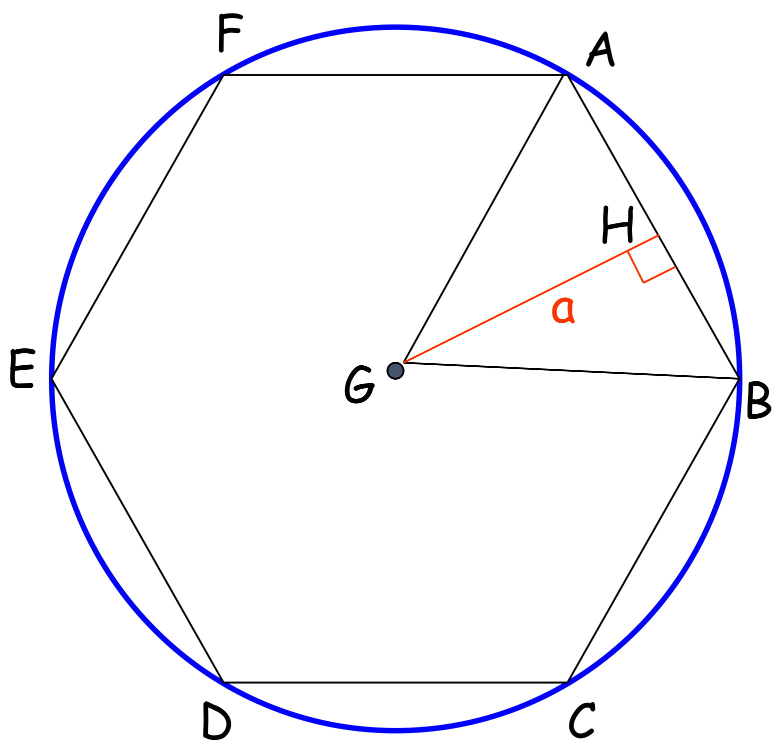
- Find the area of an equilateral triangle with 8 inch sides.

$A = \frac{1}{4}\sqrt{3}\, s^2$	Area of an equilateral Triangle
$A = \frac{1}{4}\sqrt{3}\, 8^2$	Substitute values.
$A = \frac{1}{4}\sqrt{3} \cdot 64$	Simplify.
$A = \sqrt{3} \cdot 16$	Multiply $\frac{1}{4}$ times 64.
$A = 16\sqrt{3}$	Simplify.

Using a calculator, the area is about 27.7 square inches.

- The apothem is the height of a triangle between the center and two consecutive vertices of the polygon.

- As in the activity, you can find the area o any regular n-gon by dividing the polygon into congruent triangles.



Hexagon ABCDEF with center G, radius GA, and apothem GH

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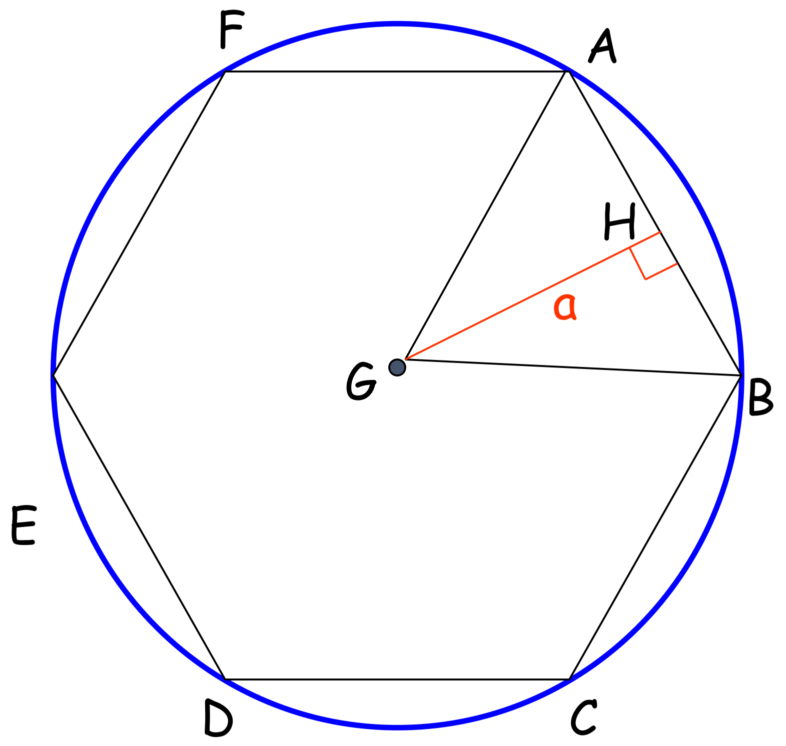
A = Area of 1 triangle • # of triangles

= (½ • apothem • side length s) • # of sides

= ½ • apothem • # of sides • side length s

= ½ • apothem • perimeter of a polygon

This approach can be used to find the area of any regular polygon.



Hexagon ABCDEF with center G, radius GA, and apothem GH

Theorem: Area of a Regular Polygon

- The area of a regular n-gon with side lengths (s) is half the product of the apothem (a) and the perimeter (P), so

  $A = \frac{1}{2} aP$, or $A = \frac{1}{2} a \bullet ns$.

The number of congruent triangles formed will be the same as the number of sides of the polygon.

NOTE: In a regular polygon, the length of each side is the same. If this length is (s), and there are (n) sides, then the perimeter P of the polygon is $n \bullet s$, or $P = ns$

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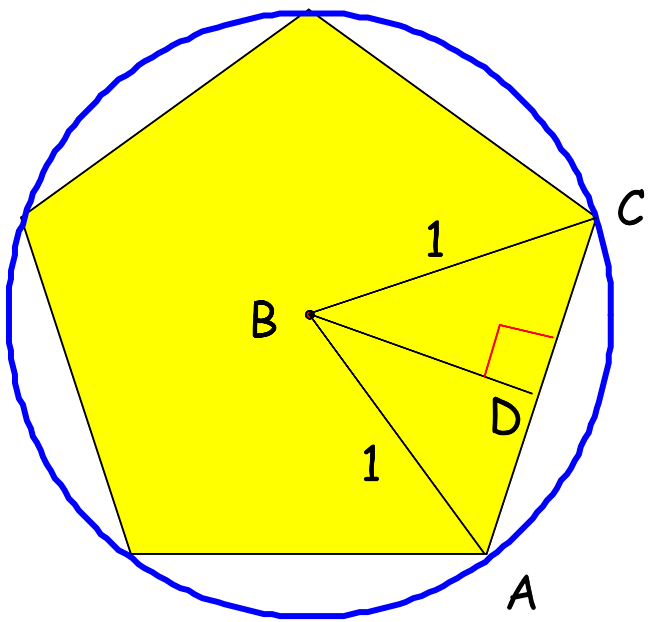
- A central angle of a regular polygon is an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon. You can divide 360° by the number of sides to find the measure of each central angle of the polygon.

- $360/n$ = central angle

Finding the area of a regular polygon

- A regular pentagon is inscribed in a circle with radius 1 unit. Find the area of the pentagon.



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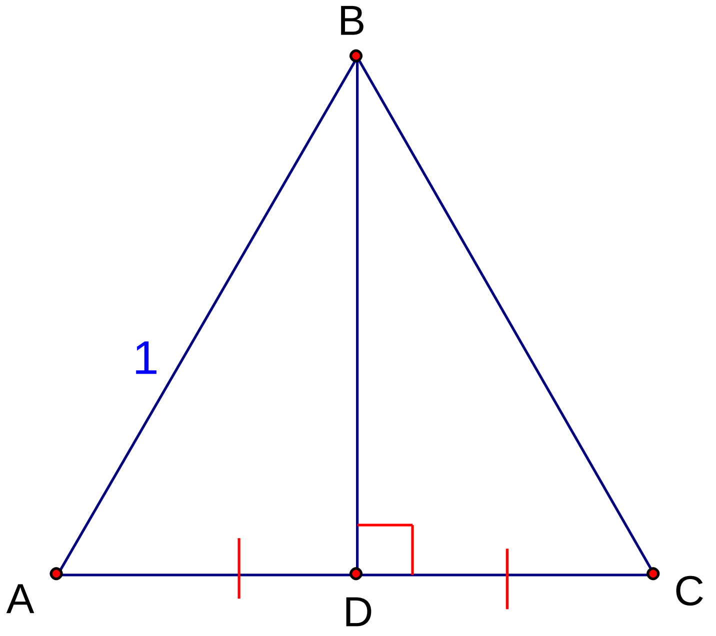
Solution:

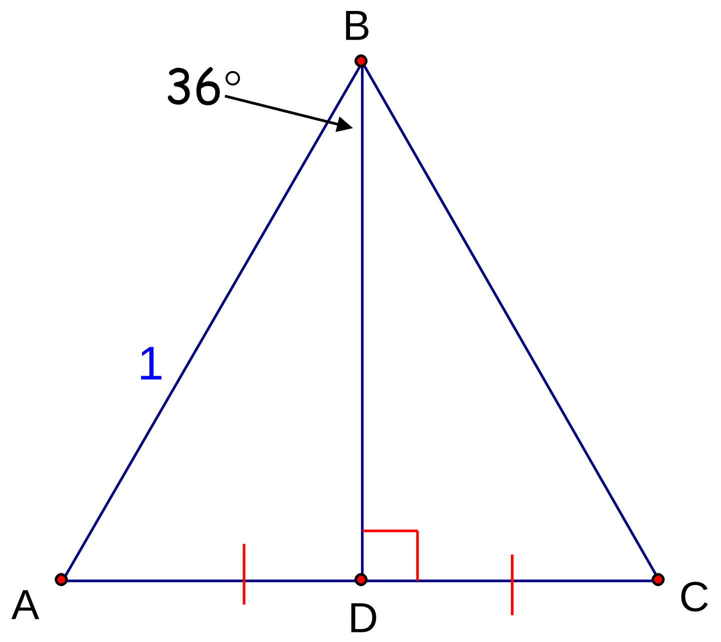
- The apply the formula for the area of a regular pentagon, you must find its apothem and perimeter.
- The measure of central ∠ABC is
  - 360°, or 72°.

$\frac{1}{5}$



Solution:

- In isosceles triangle ΔABC, the altitude to base AC also bisects ∠ABC and side AC. The measure of ∠DBC, then is 36°. In right triangle ΔBDC, you can use trig ratios to find the lengths of the legs.



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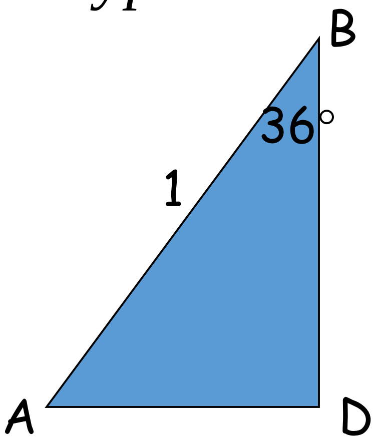
One side

- Reminder – rarely in math do you not use something you learned in the past chapters. You will learn and apply after this.

$$\sin = \frac{opp}{hyp} \qquad \cos = \frac{adj}{hyp} \qquad \tan = \frac{opp}{adj}$$

You have the hypotenuse, you know the degrees . . . use cosine



$$\cos 36^\circ = \frac{BD}{AD}$$

$$\cos 36^\circ = \frac{BD}{1}$$

$$\cos 36^\circ = BD$$

Which one?

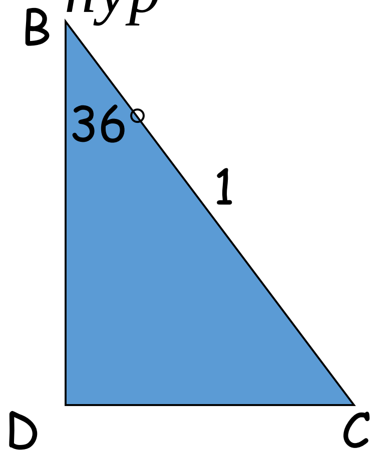
- Reminder – rarely in math do you not use something you learned in the past chapters. You will learn and apply after this.

sin = $\frac{opp}{hyp}$

cos = $\frac{adj}{hyp}$

tan = $\frac{opp}{adj}$

You have the hypotenuse, you know the degrees . . . use sine



$\sin 36^\circ = \frac{DC}{BC}$

$\sin 36^\circ = \frac{DC}{1}$

$\sin 36^\circ = DC$

SO . . .

- So the pentagon has an apothem of a = BD = cos 36° and a perimeter of P = 5(AC) = 5(2 • DC) = 10 sin 36°. Therefore, the area of the pentagon is

$A = \frac{1}{2} aP = \frac{1}{2} (\cos 36°)(10 \sin 36°) \approx 2.38$ square units.

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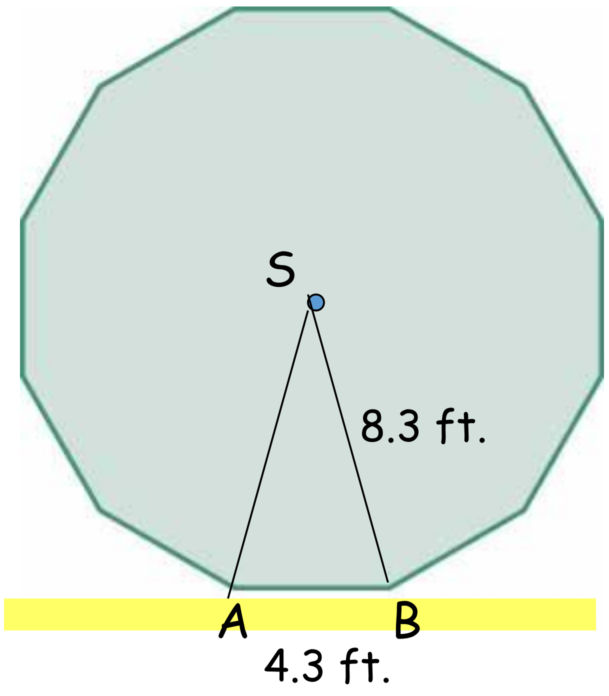
Finding the area of a regular dodecagon

- Pendulums. The enclosure on the floor underneath the Foucault Pendulum at the Houston Museum of Natural Sciences in Houston, Texas, is a regular dodecagon with side length of about 4.3 feet and a radius of about 8.3 feet. What is the floor area of the enclosure?

Solution:

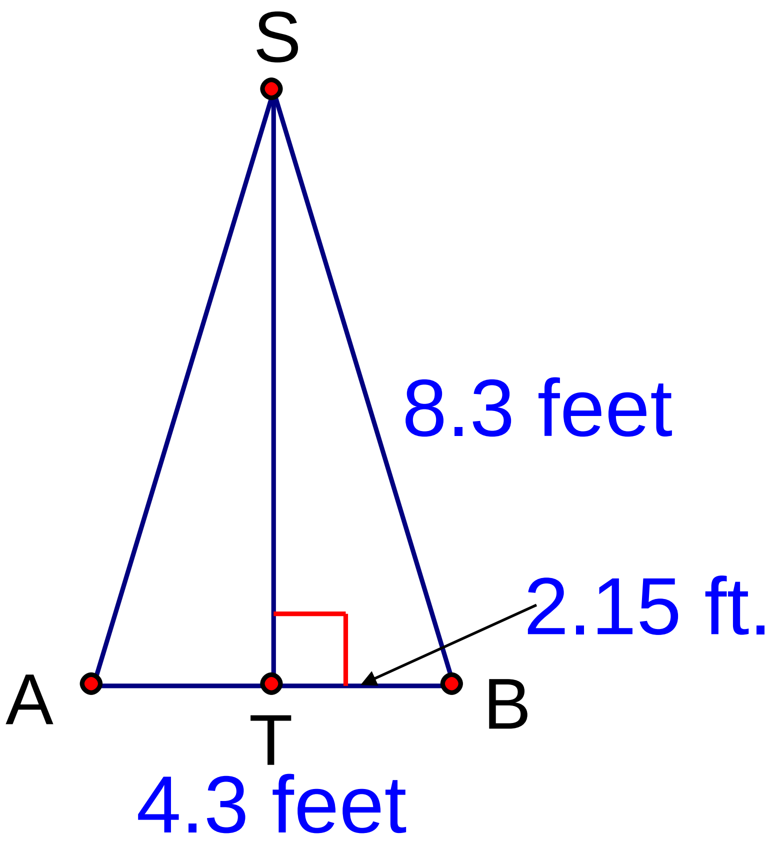
- A dodecagon has 12 sides. So, the perimeter of the enclosure is

P = 12(4.3) = 51.6 feet



Solution:

- In ΔSBT, BT = ½ (BA) = ½ (4.3) = 2.15 feet. Use the Pythagorean Theorem to find the apothem ST.



$$a = \sqrt{8.3^2 - 2.15^2}$$

$a \approx 8$ feet

So, the floor area of the enclosure is:

$A = \frac{1}{2}aP \approx \frac{1}{2}(8)(51.6) = 206.4 \text{ ft.}^2$