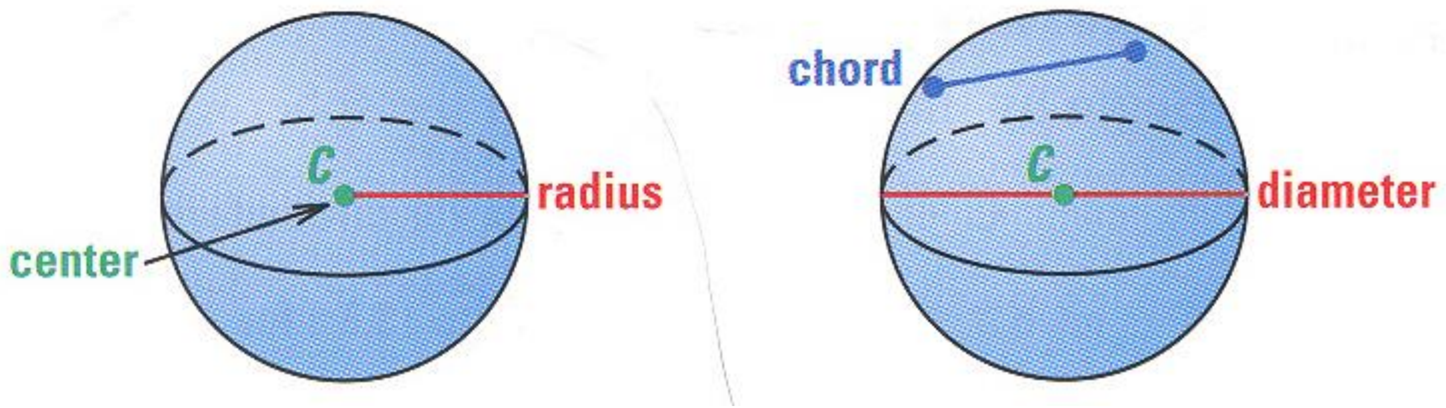
James Madison HIGH SCHOOL

Surface Area and Volume of Spheres

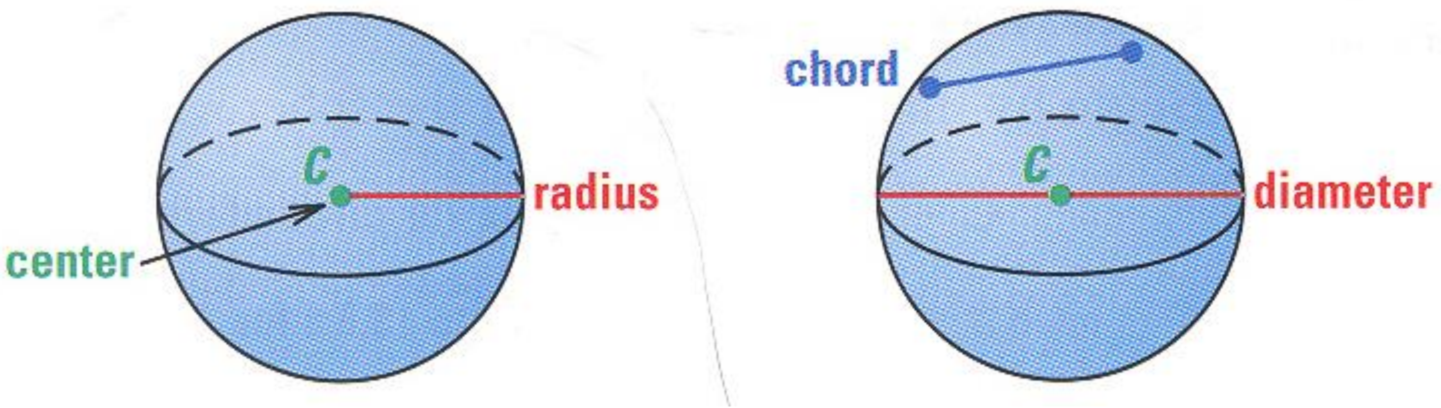
Finding the Surface Area of a Sphere

A circle is described as a locus of points in a plane that are a given distance from a point. A sphere is the locus of points in space that are a given distance from a point.



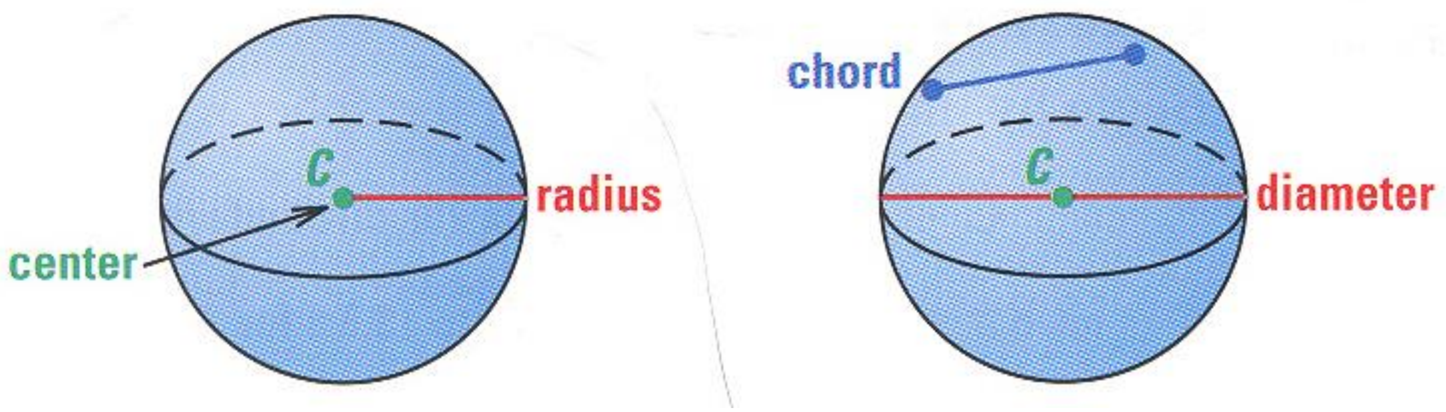
Finding the Surface Area of a Sphere

- The point is called the center of the sphere. A radius of a sphere is a segment from the center to a point on the sphere.
- A chord of a sphere is a segment whose endpoints are on the sphere.



Finding the Surface Area of a Sphere

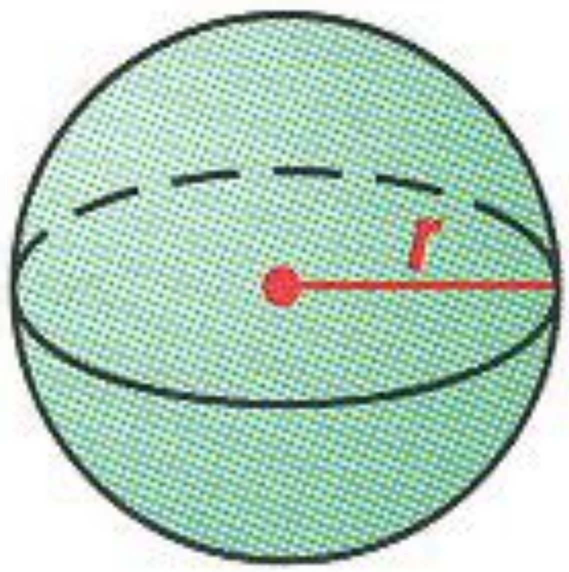
- A diameter is a chord that contains the center. As with all circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.



James Madison HIGH SCHOOL

Surface Area of a Sphere

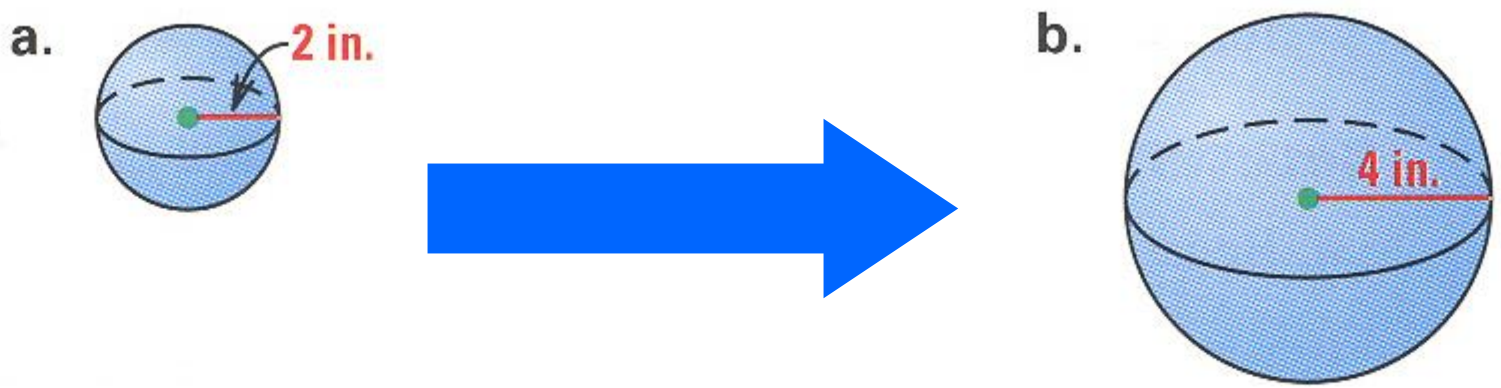
- The surface area of a sphere with radius r is $S = 4\pi r^2$.



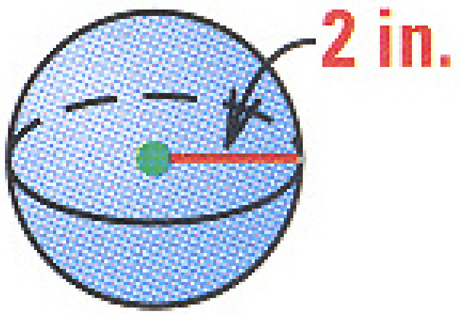
James Madison HIGH SCHOOL

Ex. 1: Finding the Surface Area of a Sphere

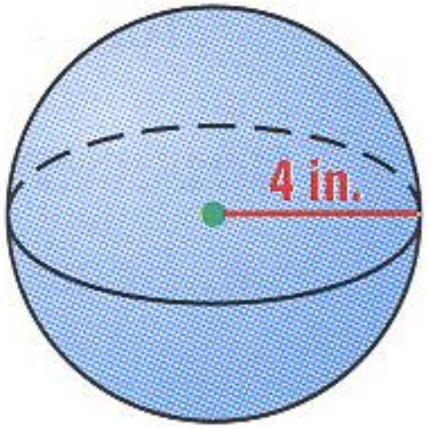
- Find the surface area. When the radius doubles, does the surface area double?



a.



b.

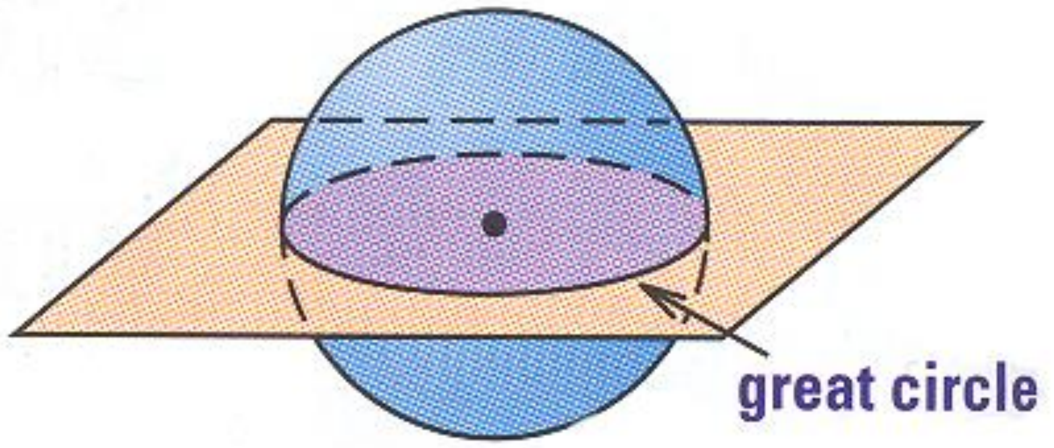


$$S = 4\pi r^2$$
$$= 4\pi 2^2$$
$$= 16\pi \text{ in.}^2$$

$$S = 4\pi r^2$$
$$= 4\pi 4^2$$
$$= 64\pi \text{ in.}^2$$

James Madison HIGH SCHOOL

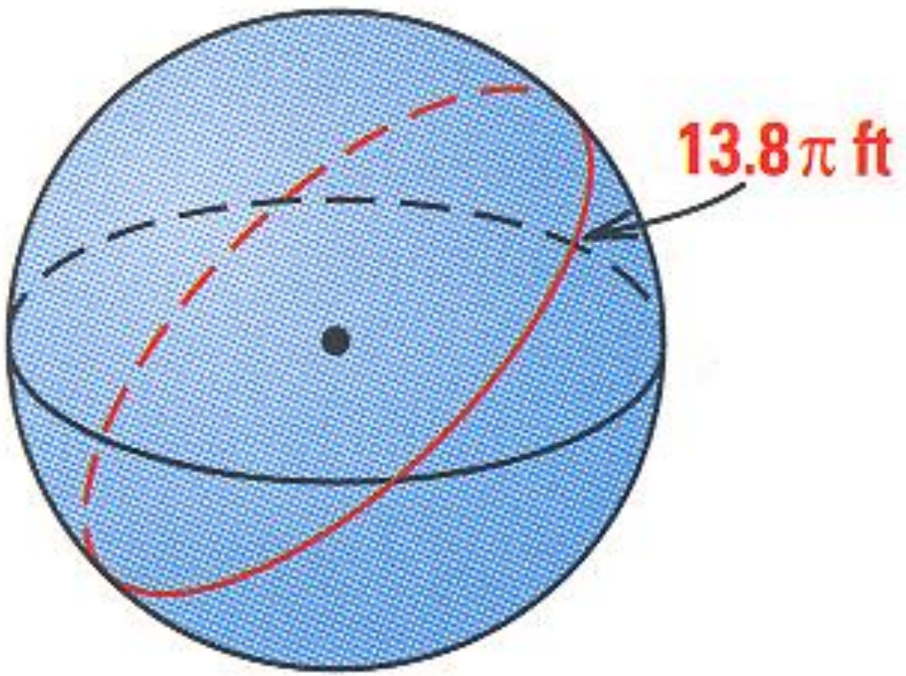
More . . .



- If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a great circle of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called hemispheres.

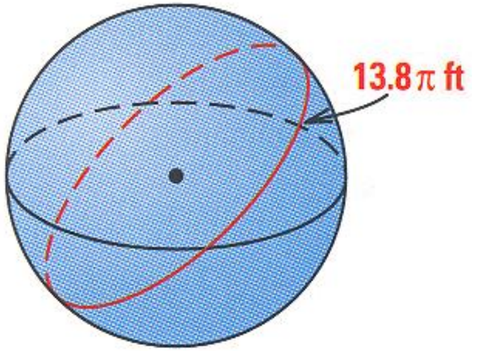
Ex. 2: Using a Great Circle

- The circumference of a great circle of a sphere is 13.8π feet. What is the surface area of the sphere?



James Madison
HIGH SCHOOL

Solution:



Begin by finding the radius of the sphere.

$C = 2\pi r$

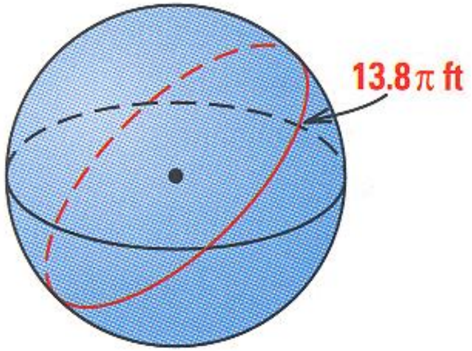
$13.8\pi = 2\pi r$

13.8π

$2\pi r$

$6.9 = r$ $= \ r$

Solution:



Using a radius of 6.9 feet, the surface area is:

$S = 4\pi r^2$

$= 4\pi(6.9)^2$

$= 190.44\pi$ ft.^{2}

So, the surface area of the sphere is 190.44 π ft.2

Ex. 3: Finding the Surface Area of a Sphere

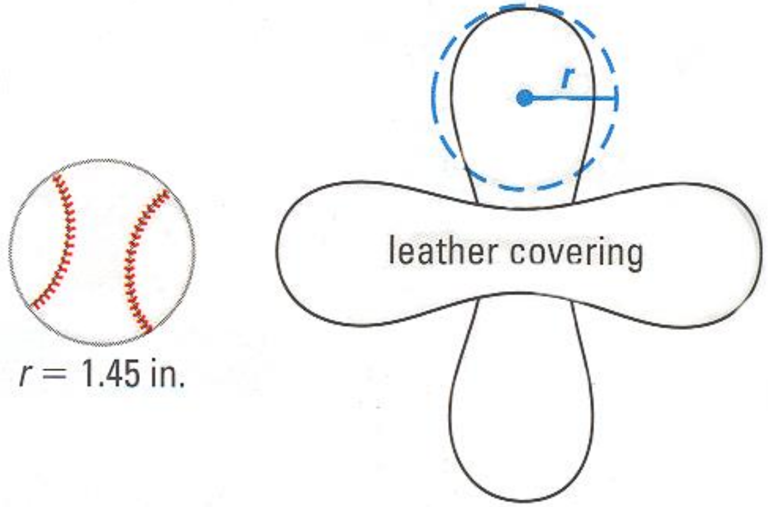
- Baseball. A baseball and its leather covering are shown. The baseball has a radius of about 1.45 inches.

a. Estimate the amount of leather used to cover the baseball.

b. The surface area of a baseball is sewn from two congruent shapes, each which resembles two joined circles. How does this relate to the formula for the surface area of a sphere?

James Madison HIGH SCHOOL

Ex. 3: Finding the Surface Area of a Sphere



SOLUTION

a. Because the radius r is about 1.45 inches, the surface area is as follows:

$$S = 4\pi r^2 \quad \text{Formula for surface area of sphere}$$

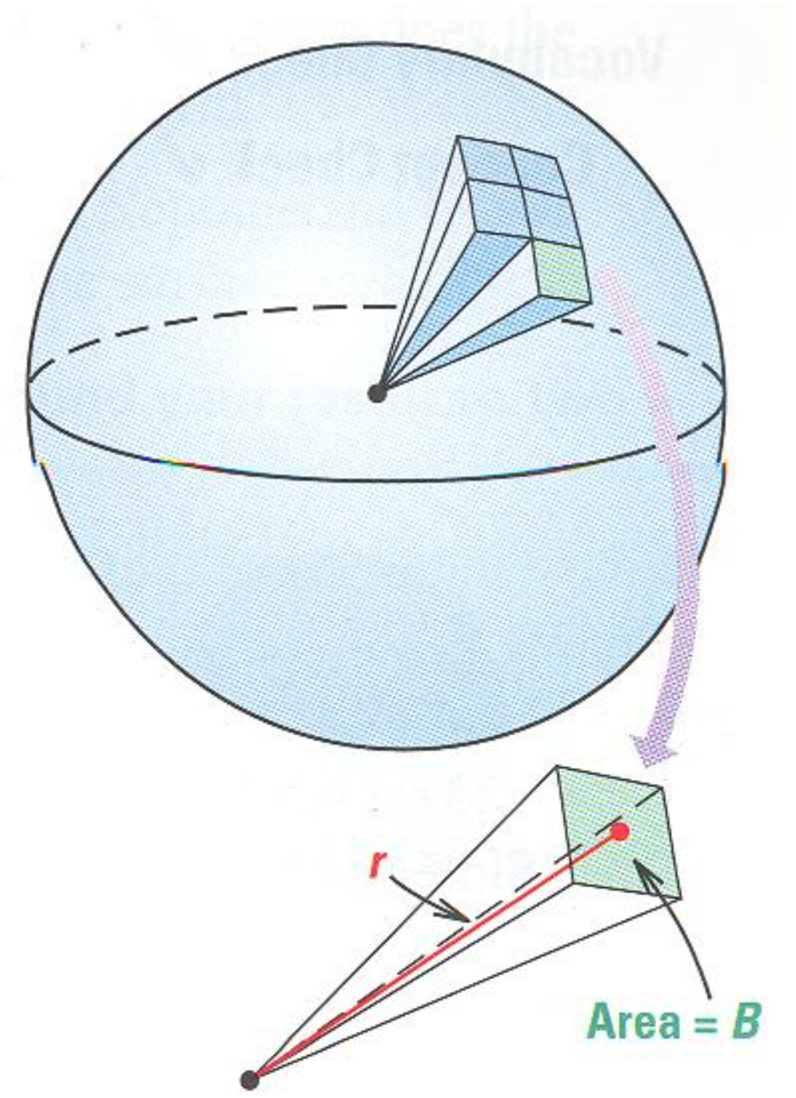
$$\approx 4\pi(1.45)^2 \quad \text{Substitute 1.45 for } r.$$

$$\approx 26.4 \text{ in.}^2 \quad \text{Use a calculator.}$$

b. Because the covering has two pieces, each resembling two joined circles, then the entire covering consists of four circles with radius r. The area of a circle of radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the same as the formula for the surface area of a sphere.

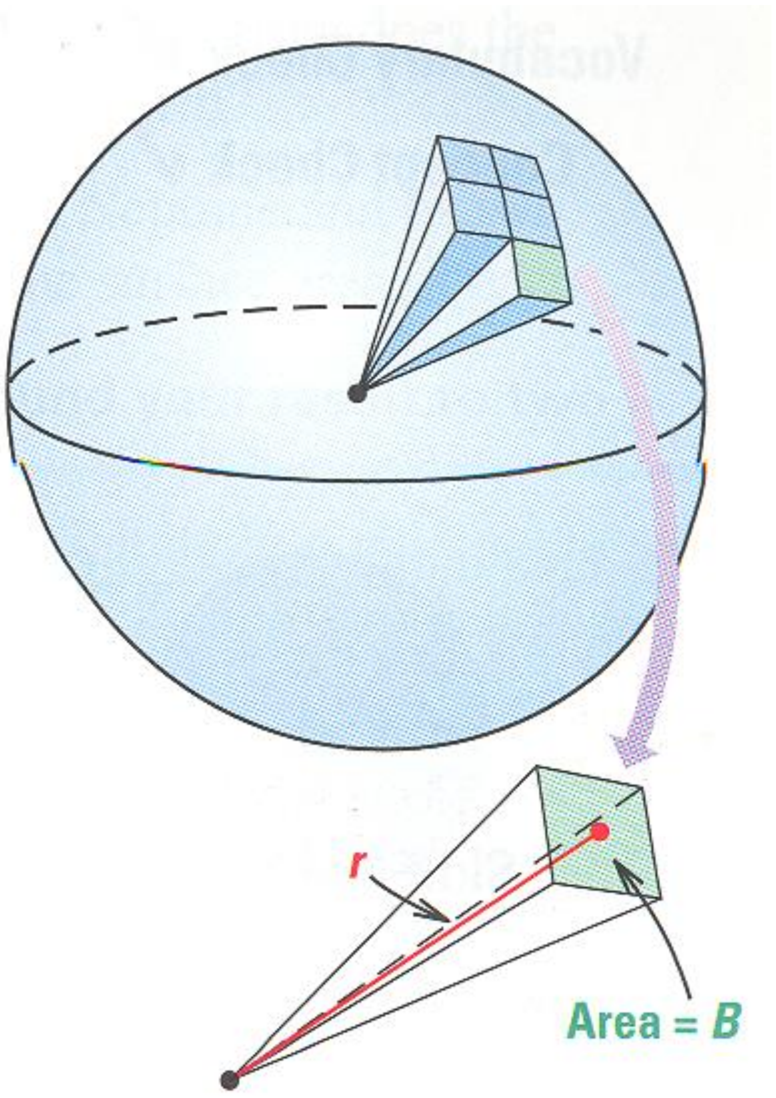
Finding the Volume of a Sphere

- Imagine that the interior of a sphere with radius r is approximated by n pyramids as shown, each with a base area of B and a height of r, as shown. The volume of each pyramid is 1/3 Br and the sum is nB.



Finding the Volume of a Sphere

- The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows:



More . . .

$V \approx n(1/3)Br$ — Each pyramid has a volume of 1/3Br.

$= 1/3\ (nB)r$ — Regroup factors.

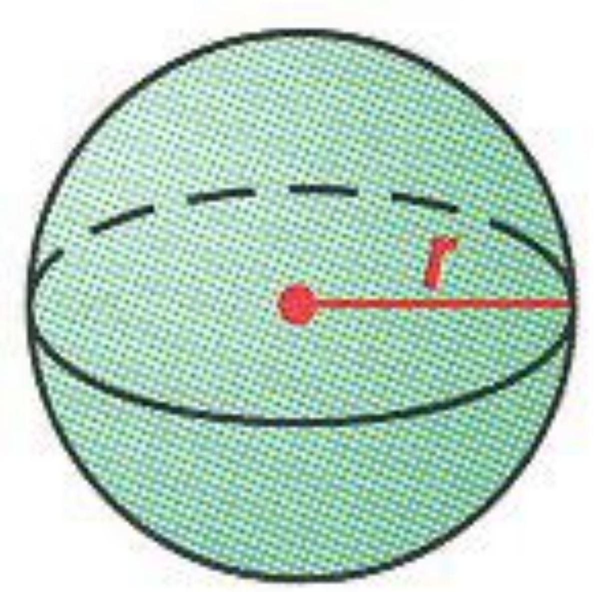
$\approx 1/3(4\pi r^2)r$ — Substitute $4\pi r^2$ for nB.

Simplify.

$= 4/3\pi r^2$

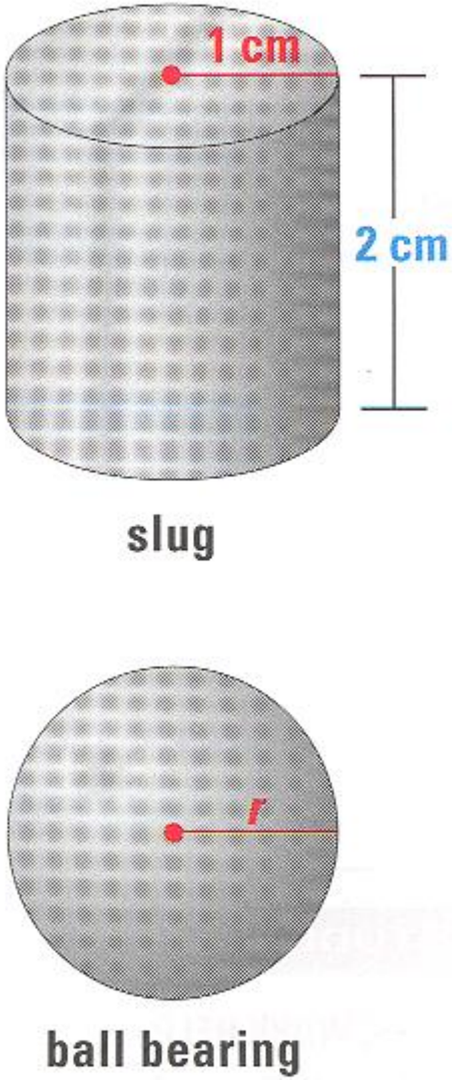
Volume of a Sphere

- The volume of a sphere with radius r is S = $4\pi r^3$.



Ex. 4: Finding the Volume of a Sphere

- Ball Bearings. To make a steel ball bearing, a cylindrical slug is heated and pressed into a spherical shape with the same volume. Find the radius of the ball bearing to the right:



James Madison HIGH SCHOOL

Solution:

- To find the volume of the slug, use the formula for the volume of a cylinder.

$V = \pi r^2 h$

$= \pi(1^2)(2)$

$= 2\pi \text{ cm}^3$

To find the radius of the ball bearing, use the formula for the volume of a sphere and solve for r.

More . . .

$V = 4/3\pi r^3$ Formula for volume of a sphere.

$2\pi = 4/3\pi r^3$ Substitute 2π for V.

$6\pi = 4\pi r^3$ Multiply each side by 3.

$1.5 = r^3$ Divide each side by 4π.

$1.14 \approx r$ Use a calculator to take the cube root.

So, the radius of the ball bearing is about 1.14 cm.