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Permutations and Combinations

There is often confusion between a permutation and a combination. Even after a teacher lectured, done in-class work, and assigned homework, students have trouble defining these terms and really understanding when to use one versus the other when modeling.

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Permutations and Combinations

This lesson will address the topics of permutations and combinations. During the lesson, students will use models to determine whether a permutation of combination is appropriate. The lesson will also enable students to determine probabilities of events using combinations.

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Show What You Already Know

1. How many possible passwords using three letters are there?
2. How many possible 3-digit numbers are there, excluding starting with zero?
3. Suppose you simultaneously roll a die and flip a coin. What is the probability of getting a one on the die and a head on the coin?

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Thoughtful Questions

1. What is a factorial and how is a factorial denoted?

2. What is zero factorial? Why?

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Permutations

Earlier we stated that there were 17,576 different 3-letter passwords possible. Suppose we were given three letters and could not use a letter more than once. Let's now determine how many 3-letter passwords are possible if no letter can be repeated.

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Permutations

Whenever we take an arrangement of objects in a specific order, we call it a permutation. Therefore, in the password problem, we determined that there were six possibilities. Now, let's calculate 3! and compare the results. What do you see? A permutation can be further simplified by using factorials.

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Permutations

We just discovered the "**Permutation of *n* Objects**". Suppose we are not given the number of letters to use in the password problem. Instead, we are asked to find the number of possible 3-letter passwords without repetition. We are required to use only three different letters, but there are 26 letters to take from. How will the process of calculation change?

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Permutations

We just discovered the "**Permutation of *n* Objects Taken *r* at a Time**". There are 26 letters, but we only wanted to use three letters. There are two ways to denote a permutation of *n* objects taken r at a time.

$$\mathbf{P}(n,r)$$

$$_{n}\mathbf{P}_{r}$$

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Permutations

A permutation involves arranging items in a specific order.

Thus,

ORDER MATTERS

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Permutations

1. How many different ways can the word **ALL** be arranged?

2. How many different ways can the word **LALL** be arranged?

3. How many ways can the word **LLALL** be arranged?

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Permutations

The previous questions involved a "**Permutation With Identical Objects**". What is a formula for this?

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Permutations

If there are three people seated at a circular table, how many different seating arrangements are possible?

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Permutations

This is called "**Circular Permutation**". What is the formula for calculating a circular permutation? How can we compare a linear permutation to a circular permutation? Let's analyze the comparison.

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What happens if order isn’t an issue? In other words, can I arrange things without worrying about the order?

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Problem

Let’s take a look at a problem. Suppose a coach of a basketball team had to choose two captains for the team. If he had four players to choose from, how many different arrangements can he have?

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Problem continued

Suppose the coach still has to choose two players from the four, but one is a captain and the other is the assistant. How many different arrangements can he have?

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Combinations

The original problem is a "**Combination of *n* Objects Taken *r* at a Time**". What is the formula for this? There are three ways to denote a combination of *n* objects taken *r* at a time.

$C(n,r)$ $\quad {}_nC_r$ $\quad \binom{n}{r}$

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Probability with Combinations

In a recent survey of 25 students, 8 opposed block scheduling and 17 favored it. Find the probability that in a random sample of 6 respondents from this survey, exactly 2 favor block scheduling and 4 oppose it.