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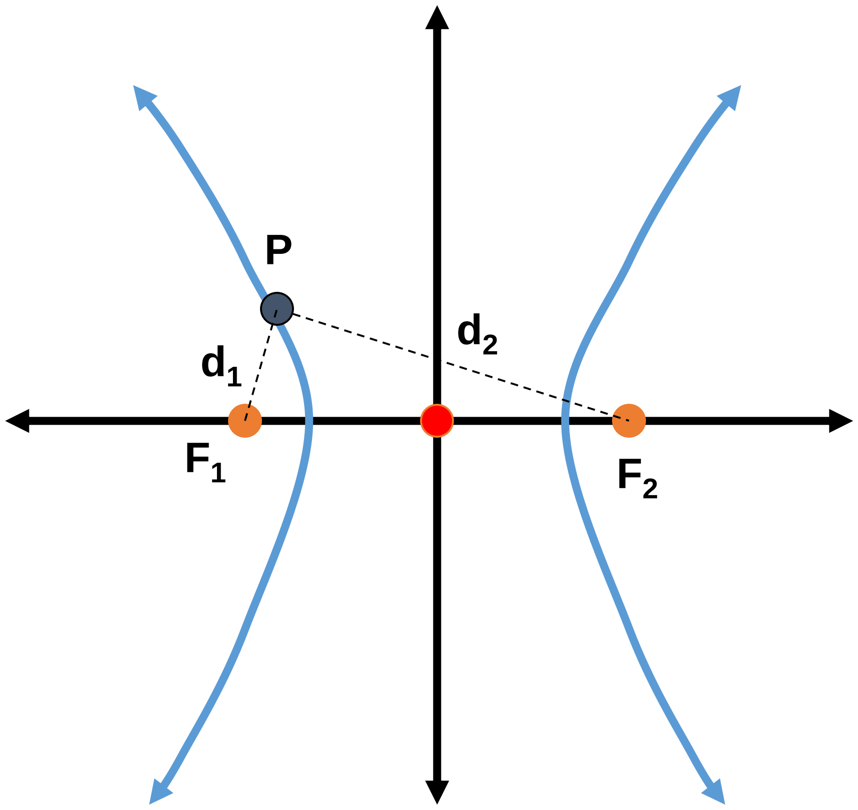
Hyperbolas

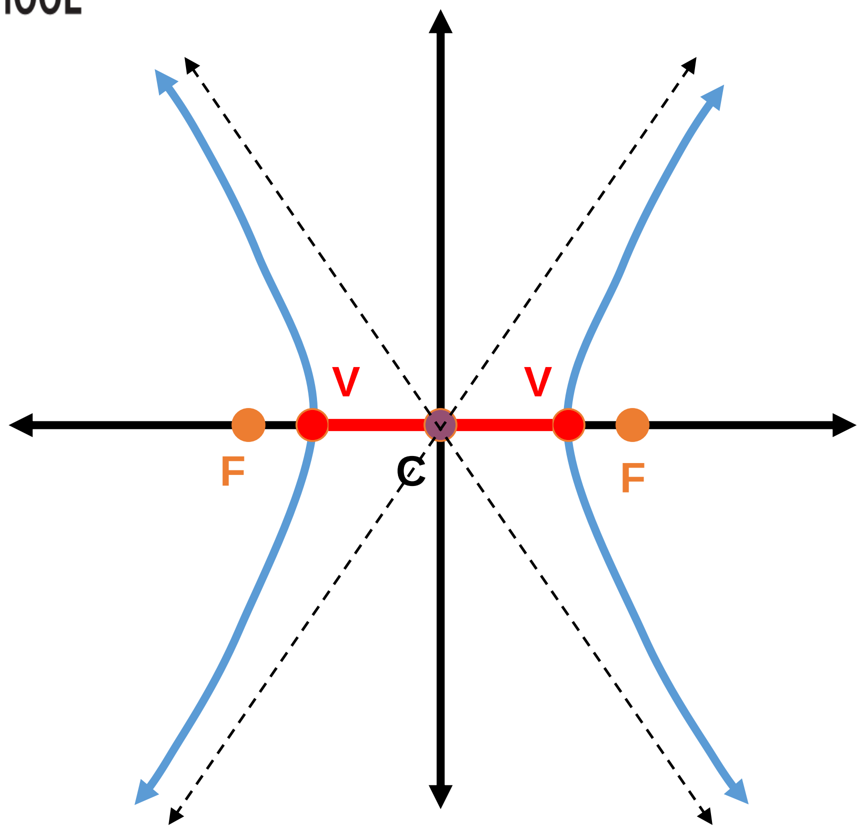
Hyperbolas

A **hyperbola** is a set of points in a plane the difference of whose distances from two fixed points, called **foci**, is a constant.

- **For any point P that is on the hyperbola, $d_2 - d_1$ is always the same.**
- **In this example, the origin is the center of the hyperbola. It is midway between the foci.**



Hyperbolas



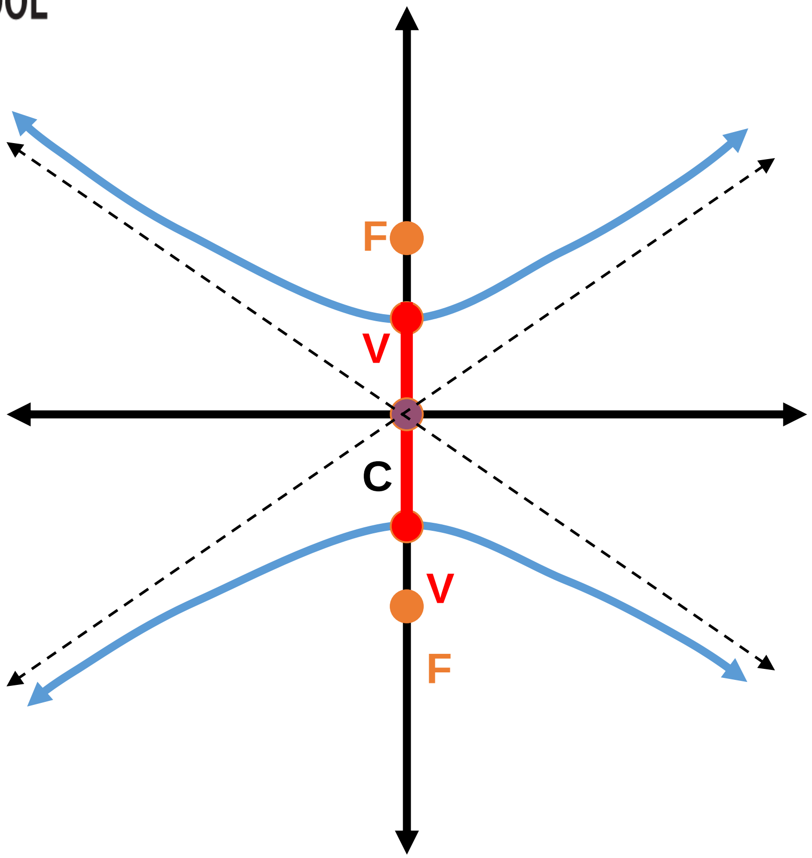
A line through the foci intersects the hyperbola at two points, called the **vertices**.

The segment connecting the vertices is called the **transverse axis** of the hyperbola.

The center of the hyperbola is located at the midpoint of the transverse axis.

As x and y get larger the branches of the hyperbola approach a pair of intersecting lines called the **asymptotes** of the hyperbola. These asymptotes pass through the center of the hyperbola.

Hyperbolas



The figure at the left is an example of a hyperbola whose branches open up and down instead of right and left.

- Since the transverse axis is vertical, this type of hyperbola is often referred to as a **vertical hyperbola**.

- When the transverse axis is horizontal, the hyperbola is referred to as a **horizontal hyperbola**.

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Standard Form Equation of a Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Horizontal Hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Vertical Hyperbola

★ **The center of a hyperbola is at the point (h, k) in either form**

★ **For either hyperbola,** $c^2 = a^2 + b^2$

Where c is the distance from the center to a focus point.

★ **The equations of the asymptotes are**

$$y = \frac{b}{a}(x-h) + k \quad \text{and} \quad y = \frac{-b}{a}(x-h) + k$$

Graphing a Hyperbola

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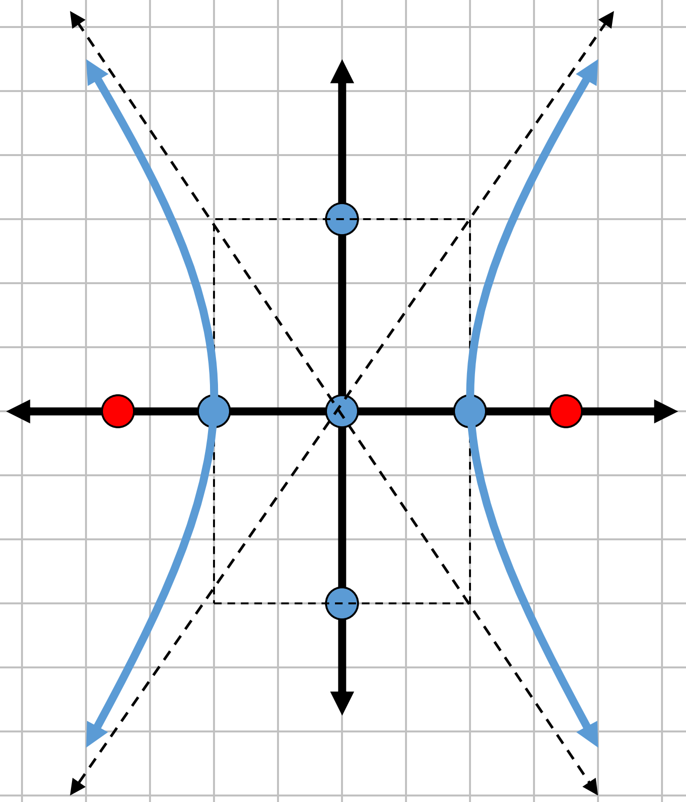
Graph: $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Center: (0, 0)

- ☆ **The x-term comes first in the subtraction so this is a horizontal hyperbola**
- ☆ **From the center locate the points that are two spaces to the right and two spaces to the left**
- ☆ **From the center locate the points that are up three spaces and down three spaces**
- ☆ **Draw a dotted rectangle through the four points you have found.**
- ☆ **Draw the asymptotes as dotted lines that pass diagonally through the rectangle.**
- ☆ **Draw the hyperbola.**



$c^2 = 9 + 4 = 13$

$c = \sqrt{13} = 3.61$

Vertices: (2, 0) and (-2, 0)

Foci: (3.61, 0) and (-3.61, 0)

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Graphing a Hyperbola

Graph: $\frac{(x+2)^2}{9} - \frac{(y-1)^2}{25} = 1$

Horizontal hyperbola

Center: (-2, 1)

Vertices: (-5, 1) and (1, 1)

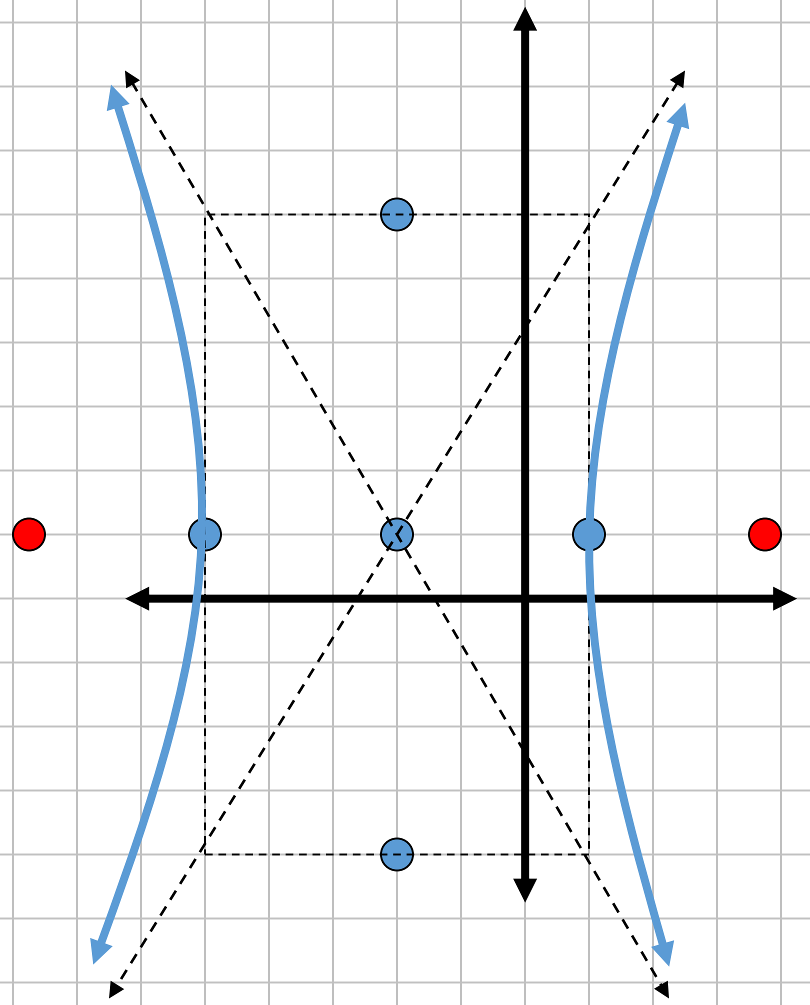
$c^2 = 9 + 25 = 34$

$c = \sqrt{34} = 5.83$

Foci: (-7.83, 1) and (3.83, 1)

Asymptotes: $y = \frac{5}{3}(x+2) + 1$

$y = -\frac{5}{3}(x+2) + 1$



Converting an Equation

Graph: $9y^2 - 4x^2 - 18y + 24x - 63 = 0$

$9(y^2 - 2y + \underline{\ 1\ }) - 4(x^2 - 6x + \underline{\ 9\ }) = 63 + \underline{\ 9\ } - \underline{36}$

$9(y - 1)^2 - 4(x - 3)^2 = 36$

$$\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1$$

The hyperbola is vertical

Center: (3, 1)

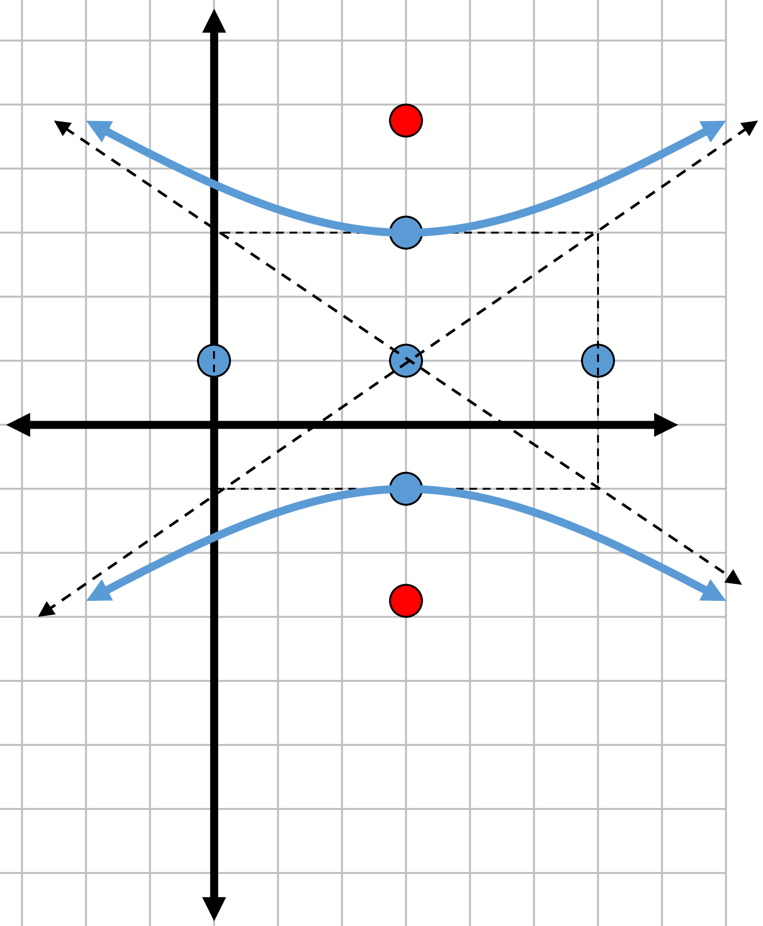
$c^2 = 9 + 4 = 13$

$c = \sqrt{13} = 3.61$

Foci: (3, 4.61) and (3, -2.61)

Asymptotes: $y = \frac{2}{3}(x-3) + 1$

$y = -\frac{2}{3}(x-3) + 1$



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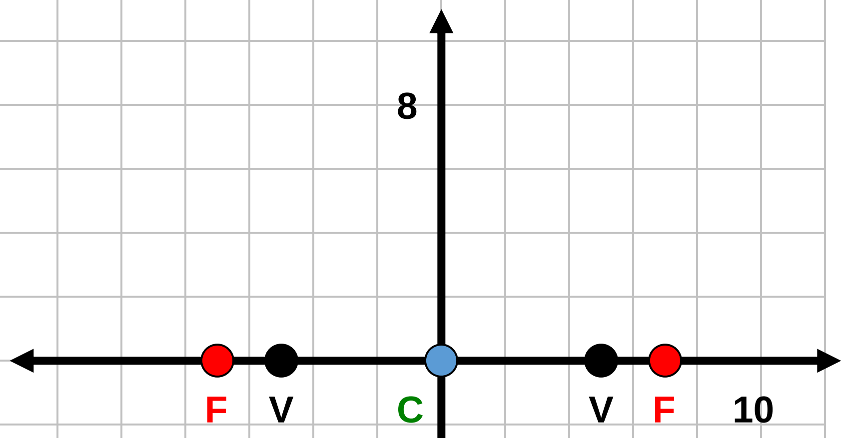
Finding an Equation

Find the standard form of the equation of a hyperbola given:

Foci: (-7, 0) and (7, 0)

Vertices: (-5, 0) and (5, 0)



Horizontal hyperbola

Center: (0, 0)

$a^2 = 25$ and $c^2 = 49$

$c^2 = a^2 + b^2$

$49 = 25 + b^2$

$b^2 = 24$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

Finding an Equation

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Find the standard form equation of the hyperbola that is graphed at the right

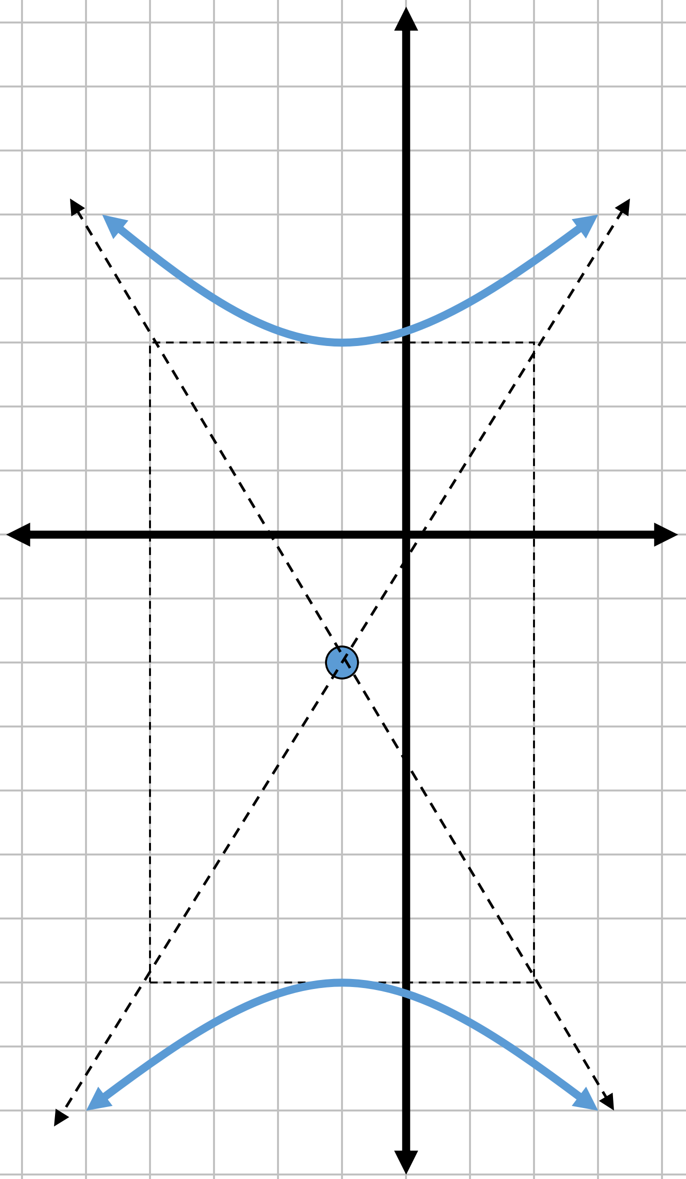
Vertical hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Center: (-1, -2)

a = 3 and b = 5

$$\frac{(y+2)^2}{25} - \frac{(x+1)^2}{9} = 1$$



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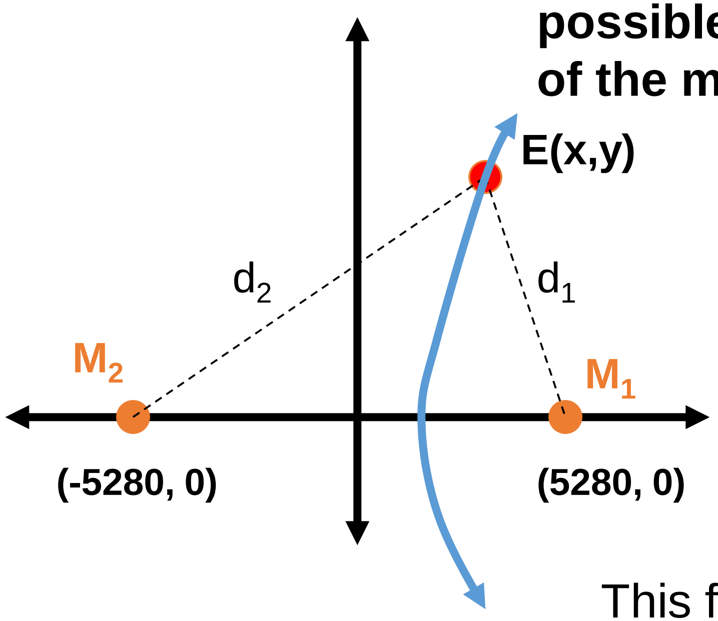
Applications

An explosion is recorded by two microphones that are two miles apart. M1 received the sound 4 seconds before M2. assuming that sound travels at 1100 ft/sec, determine the possible locations of the explosion relative to the locations of the microphones.



Let us begin by establishing a coordinate system with the origin midway between the microphones

Since the sound reached M_2 4 seconds after it reached M_1, the difference in the distances from the explosion to the two microphones must be

1100(4) = 4400 ft wherever E is

This fits the definition of an hyperbola with foci at M_1 and M_2

$$\frac{x^2}{a_2} - \frac{y^2}{b_2} = 1$$

Since $d_2 - d_1$ = transverse axis, a = 2200

$$c^2 = a^2 + b^2$$

$$5280^2 = 2200^2 + b^2$$

$$b^2 = 23{,}038{,}400$$

The explosion must by on the hyperbola

$$\frac{x^2}{4{,}840{,}000} - \frac{y^2}{23{,}038{,}400} = 1$$