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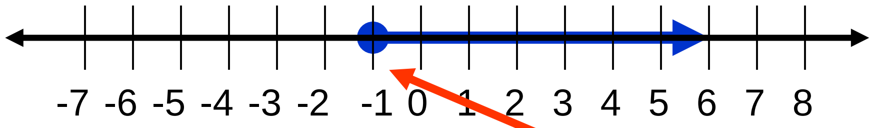
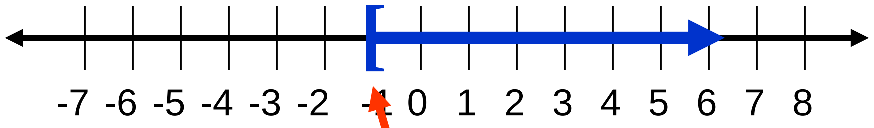
Linear Inequalities

Remember---these mean the same thing---just two different notations.

familiar with both.

$$x \geq -1$$





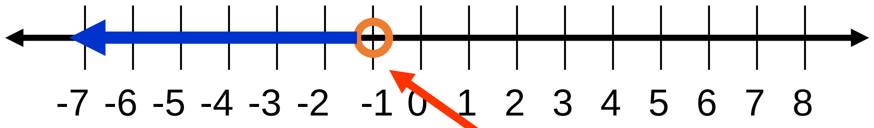
circle filled in

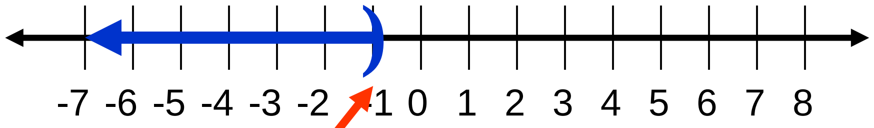
squared end bracket

Both of these number lines show the inequality above. They are just using two different notations. Because the inequality is "greater than ***or equal to"*** the solution can equal the endpoint. That is why the circle is filled in. With interval notation brackets, a square bracket means it can equal the endpoint.

Remember---these mean the same thing---just two different notations.

$$x < -1$$





circle not filled in

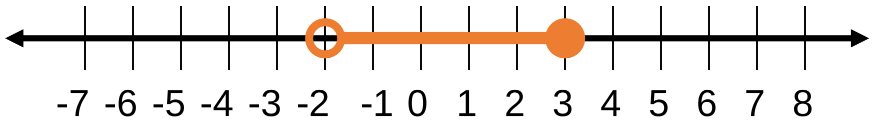
rounded end bracket

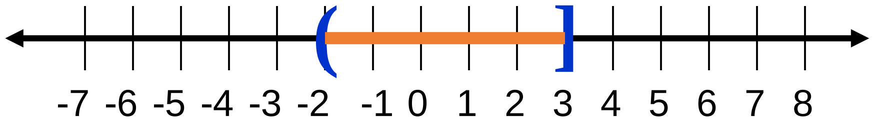
Since this says "less than" we make the arrow go the other way. Since it doesn't say "***or equal to***" the solution cannot equal the endpoint. That is why the circle is not filled in. With interval notation brackets, a rounded bracket means it cannot equal the endpoint.

Compound Inequalities

Let's consider a "double inequality" (having two inequality signs).

$$-2 < x \leq 3$$



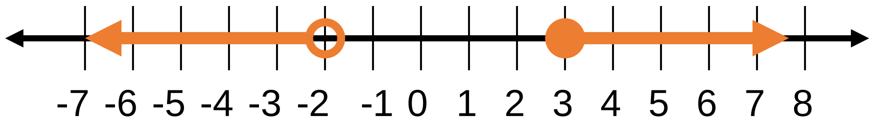


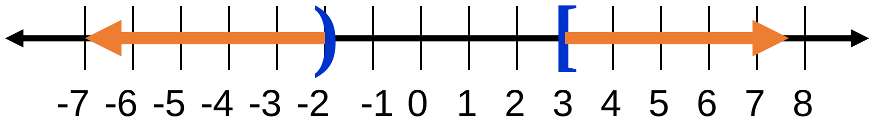
I think of these as the "inbetweeners".
x is inbetween the two numbers. This is an "and" inequality which means <u>both parts must be true</u>. It says that *x* is greater than –2 and *x* is less than or equal to 3.

Compound Inequalities

Now let's look at another form of a "double inequality" (having two inequality signs).

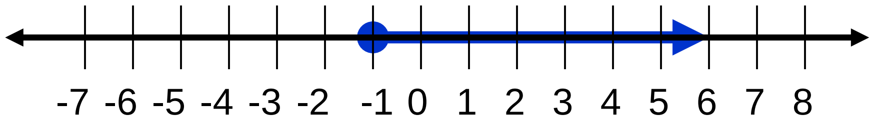
$$x < -2 \text{ or } x \geq 3$$

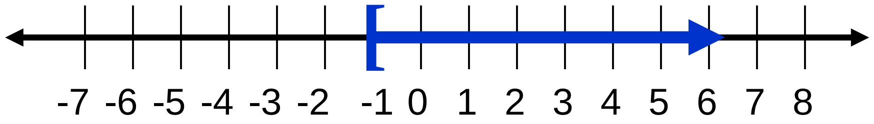




Instead of "and", these are "or" problems. One part or the other part must be true (but not necessarily both). Either *x* is less than –2 or *x* is greater than or equal to 3. In this case both parts cannot be true at the same time since a number can't be less than –2 and also greater than 3.

Just like graphically there are two different notations, when you write your answers you can use inequality notation or interval notation.
Again you should be familiar with both.





$$x \geq -1$$

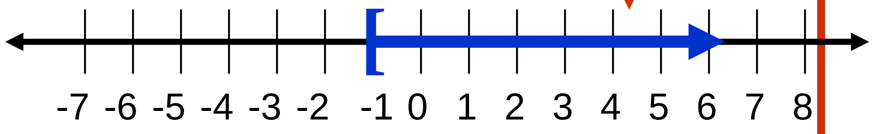
Inequality notation for graphs shown above.

$$[-1, \infty)$$

Interval notation for graphs shown above.

Let's have a look at the interval notation.

$[-1, \infty)$



This means *x* is **unbounded** above

For interval notation you list the smallest *x* can be, a comma, and then the largest *x* can be so solutions are anything that falls between the smallest and largest.

The bracket before the –1 is square because this is greater than "or equal to" (solution can equal the endpoint).

The bracket after the infinity sign is rounded because the interval goes on forever (unbounded) and since infinity is not a number, it doesn't equal the endpoint (there is no endpoint).

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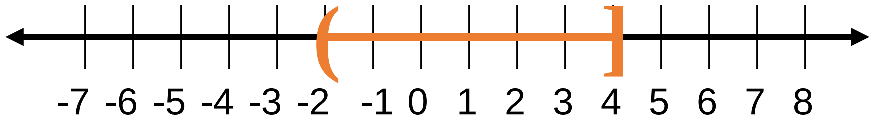
Let's try another one.

Rounded bracket means **cannot** equal -2

$(-2,4]$

Squared bracket means **can** equal 4

The brackets used in the interval notation above are the same ones used when you graph this.



This means everything between –2 and 4 but not including -2

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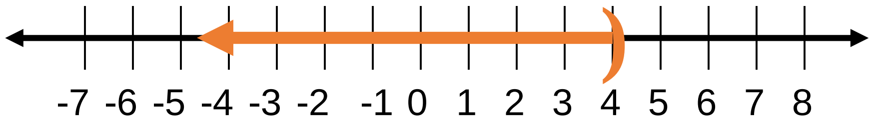
Let's look at another one

$$(-\infty, 4)$$

This means *x* is unbounded below

This means the largest *x* can be is 4 but can't equal 4



Notice how the bracket notation for graphing corresponds to the brackets in interval notation.

Remember that square is "or equal to" and round is up to but not equal. By the infinity sign it will always be round because it can't equal infinity (that is not a number).

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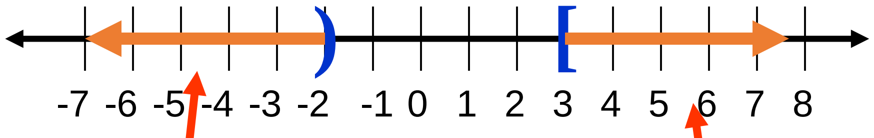
Now let's look an "or" compound inequality

$$x < -2 \text{ or } x \geq 3$$



$$(-\infty, -2) \cup [3, \infty)$$

When the solution consists of more than one interval, we join them with a union sign.

There are two intervals to list when you list in interval notation.

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Essentially, all of the properties that you learned to solve linear equations apply to solving linear inequalities with the exception that if you multiply or divide by a negative you must reverse the inequality sign.

So to solve an inequality just do the same steps as with an equality to get the variable alone but if in the process you multiply or divide by a negative let it ring an alarm in your brain that says "Oh yeah, I have to turn the sign the other way to keep it true".

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Example: $2x - 6 < 4x + 8$

$- 4x \qquad - 4x$

$-2x - 6 < 8$

$+6 \qquad +6$

$$\frac{-2x}{-2} < \frac{14}{-2}$$

Ring the alarm! We divided by a negative!

We turned the sign!

$x > -7$