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Solving Equations

- Variables on both sides
- Equations with Extraneous Roots
- Solving Quadratic Equations
- Completing the Square
- Using the Quadratic Formula
- Factoring
- Grouping

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Solving Equations

Ex. 1 **Solve.** $6(x - 1) + 4 = 7x + 1$

$x = -3$

Ex. 2 $\frac{x}{3} + \frac{3x}{4} = 2$

Multiply each term by the LCD -----> 12

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2$$

$4x + 9x = 24$

$13x = 24$

$$x = \frac{24}{13}$$

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An equation with an Extraneous Solution

$$\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$$

Again, mult. Each term by the LCD.

$$\frac{1}{x-2}(x+2)(x-2) = \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

x + 2 = 3(x - 2) - 6x

x + 2 = 3x - 6 - 6x

4x = -8

x = -2

But, x = -2 yields a den. of zero, therefore there are no solutions.

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Solving a Quadratic by Factoring

A. $6x^2 = 3x$

First, take the 3x to the other side and then factor.

$6x^2 - 3x = 0$

$3x(2x - 1) = 0$ Set both factors = to 0.

$3x = 0$ $\quad$ $2x - 1 = 0$

$x = 0$ $\quad$ $2x = 1$

$x = 1/2$

B. $9x^2 - 6x + 1 = 0$ First, factor.

$(3x - 1)(3x - 1) = 0$ Set = to 0 and solve.

$$x = \frac{1}{3}$$

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Solving quadratic equations.

A. $4x^2 = 12$

$x^2 = 3$

$x = \pm\sqrt{3}$

B. $(x - 2)^2 = 5$

$x - 2 = \pm\sqrt{5}$

$x = 2 \pm \sqrt{5}$

Completing the Square

$x^2 - 6x + 2 = 0$

First, take 2 to the other side.

$x^2 - 6x = -2$

To complete the square take half the x-term and square it. Add it to both sides.

$x^2 - 6x + 9 = -2 + 9$

$(x - 3)^2 = 7$

$x - 3 = \pm\sqrt{7}$

$x = 3 \pm \sqrt{7}$

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Completing the Square when the leading coefficient is not 1

$3x^2 - 4x - 5 = 0$ **Divide each term by 3.**

$x^2 - \frac{4x}{3} - \frac{5}{3} = 0$ **Take 5/3 to the other side.**

$x^2 - \frac{4x}{3} = \frac{5}{3}$ **Now, complete the square.**

$x^2 - \frac{4x}{3} + \left(\frac{2}{3}\right)^2 = \frac{5}{3} + \left(\frac{2}{3}\right)^2$ **Take the square root of both sides.**

$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9}$ $\left(x - \frac{2}{3}\right) = \pm\sqrt{\frac{19}{9}}$ $x = \frac{2}{3} \pm \frac{\sqrt{19}}{3}$

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Use the Quadratic Formula

$$x^2 + 3x - 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{45}}{2} \qquad x = \frac{-3 \pm 3\sqrt{5}}{2}$$

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Solve by factoring.

$x^4 - 3x^2 + 2 = 0$ **Factor**

$(x^2 - 2)(x^2 - 1) = 0$

$x^2 - 2 = 0$ $x^2 - 1 = 0$ Set both factors = 0 or factor again.

$x^2 = 2$ $x^2 = 1$

$x = \pm\sqrt{2}$ $x = \pm 1$

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Solve by grouping.

$x^3 - 3x^2 - 3x + 9 = 0$

$x^2(x - 3) - 3(x - 3) = 0$ **Factor out an (x - 3)**

$(x - 3)(x^2 - 3) = 0$

$x = 3$ $x = \pm\sqrt{3}$

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Solving a Radical

$$\sqrt{2x+7} - x = 2$$

Isolate the radical.

$$\sqrt{2x+7} = x+2$$

Now square both sides.

$$2x + 7 = x^2 + 4x + 4$$

$$0 = x^2 + 2x - 3$$

Factor or use quad. formula

$$0 = (x + 3)(x - 1)$$

Possible answers for x are - 3 and 1.
Check them in the original equation to see if they work.

Only x = 1 works!

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An equation Involving Two Radicals

$$\sqrt{2x+6}-\sqrt{x+4}=1$$

Isolate the more complicated rad.

$$\sqrt{2x+6}=\sqrt{x+4}+1$$

Square both sides.

$$2x+6=(x+4)+2\sqrt{x+4}+1$$

Once again, isolate the radical.

$$x+1=2\sqrt{x+4}$$

Square both sides.

$x^2 + 2x + 1 = 4(x + 4)$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

Only x = 5 works.

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Solving an Equation Involving Absolute Value

$$\left|x^2 - 3x\right| = -4x + 6$$

Split into 2 equations.

$x^2 - 3x = -4x + 6$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x^2 - 3x = 4x - 6$

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

Now solve them for x.

Possible answers are -3, 2, 1, and 6. Which ones work?

-3 and 1