



**James Madison**  
HIGH SCHOOL

# Functions and Their Graphs

## Key Definitions

Function - a relationship between values: each of its input values( $x$ ) gives back exactly one output value( $y$ )

Domain - set of all input values( $x$ )

Range - set of all output values( $y$ )



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Tell whether the equations represent  $y$  as a function of  $x$ .

a.  $x^2 + y = 1$

$$y = 1 - x^2$$

No, so this equation is a function.

Solve for  $y$ .

For every number we plug in for  $x$ , do we get more than one  $y$  out?

b.  $-x + y^2 = 1$

$$y^2 = x + 1$$

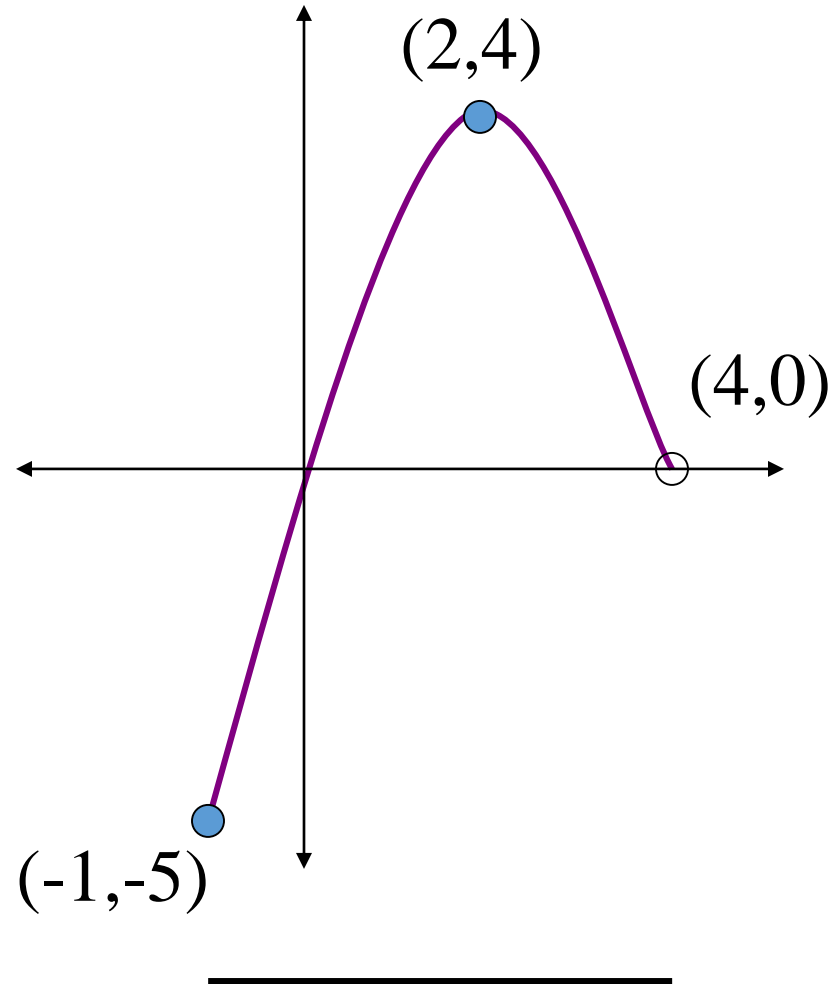
$$y = \pm\sqrt{1 + x}$$

Solve for  $y$ .

Here we have 2  $y$ 's for each  $x$  that we plug in. Therefore, this equation is not a function.

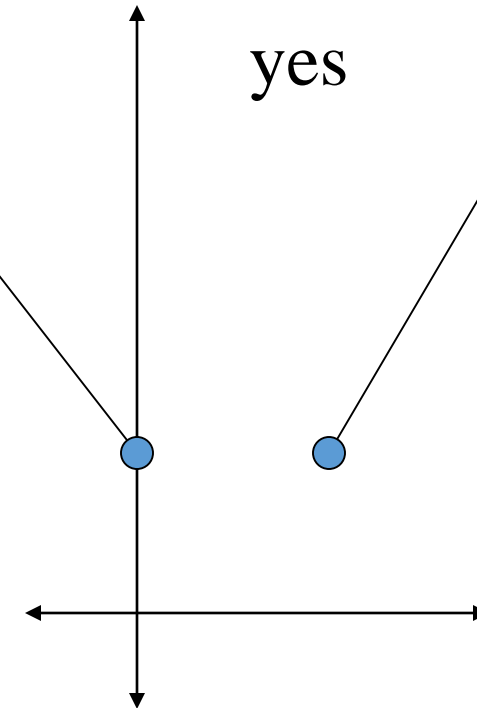
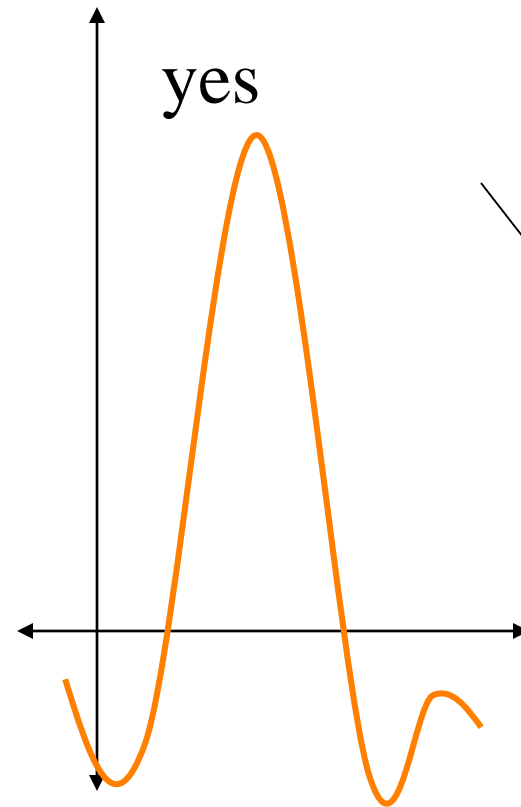
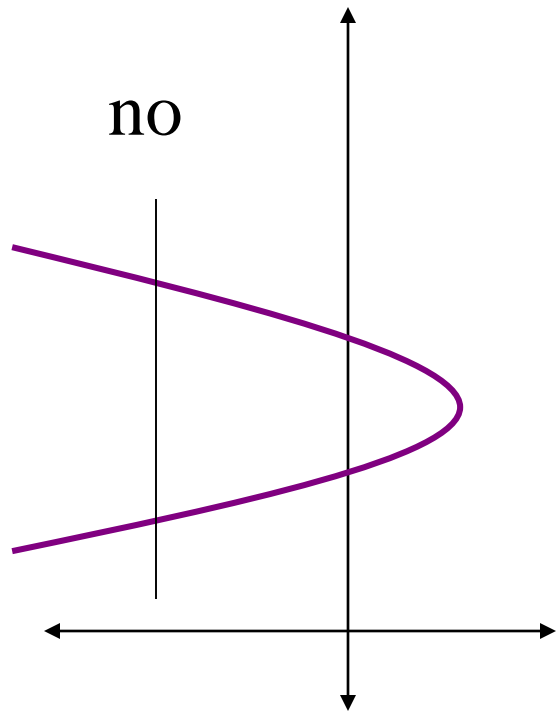
Find:

- the domain  
 $[-1,4)$
- the range  
 $[-5,4]$
- $f(-1) = -5$
- $f(2) = 4$



## Vertical Line Test for Functions

Do the graphs represent  $y$  as a function of  $x$ ?





## Tests for Even and Odd Functions

A function is  $y = f(x)$  is even if, for each  $x$  in the domain of  $f$ ,

$$f(-x) = f(x)$$

An even function is symmetric about the  $y$ -axis.

A function is  $y = f(x)$  is odd if, for each  $x$  in the domain of  $f$ ,

$$f(-x) = -f(x)$$

An odd function is symmetric about the origin.

Ex.  $g(x) = x^3 - x$

$$g(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x)$$

Therefore,  $g(x)$  is odd because  $f(-x) = -f(x)$

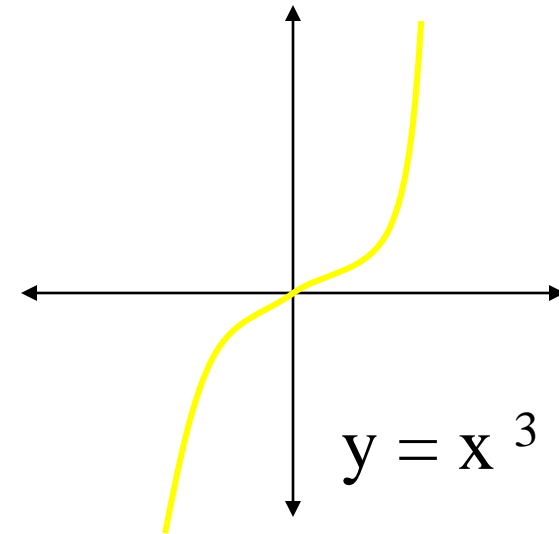
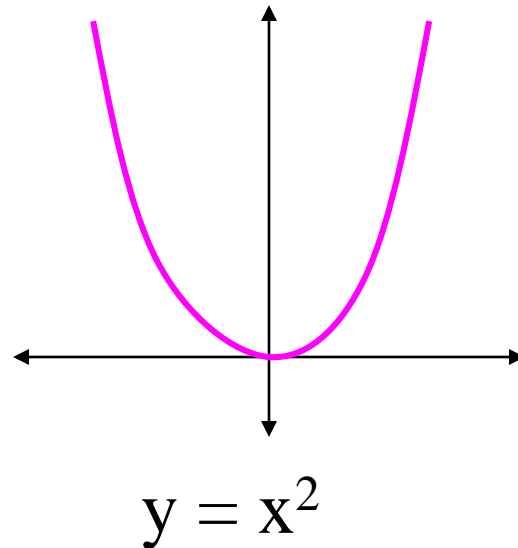
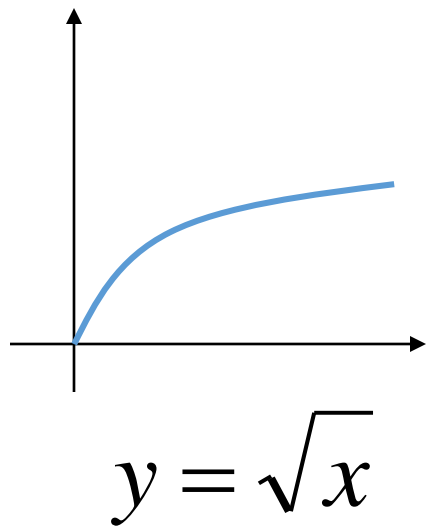
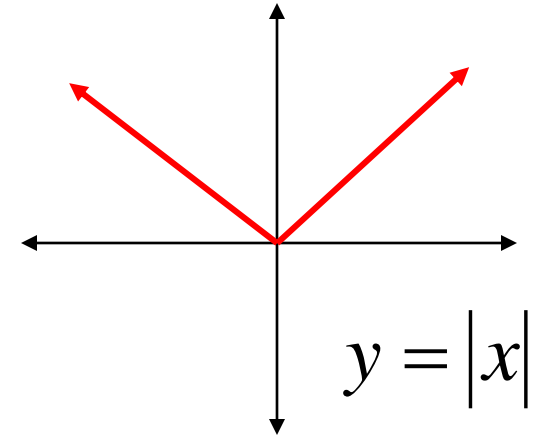
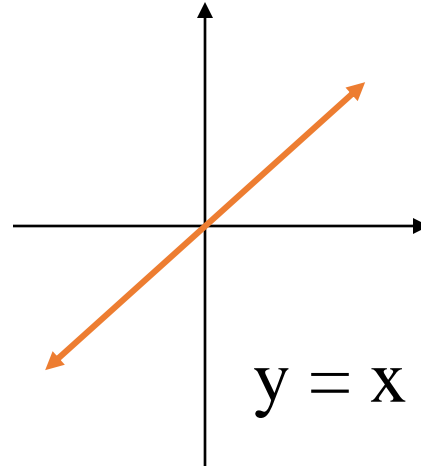
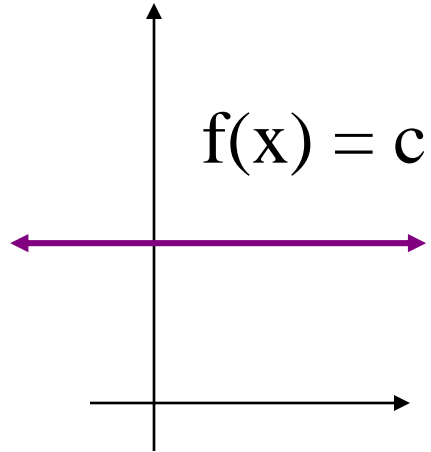
Ex.  $h(x) = x^2 + 1$

$$h(-x) = (-x)^2 + 1 = x^2 + 1$$

$h(x)$  is even because  $f(-x) = f(x)$



# Summary of Graphs of Common Functions





## Vertical and Horizontal Shifts

1.  $h(x) = f(x) + c$       Vert. shift up
2.  $h(x) = f(x) - c$       Vert. shift down
3.  $h(x) = f(x - c)$       Horiz. shift right
4.  $h(x) = f(x + c)$       Horiz. shift left
5.  $h(x) = -f(x)$       Reflection in the x-axis
6.  $h(x) = f(-x)$       Reflection in the y-axis