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Inverse Functions

Definition of Inverse: A function g is the inverse of the function f if f(g(x)) = x and g(f(x)) = x.

Domain of f = Range of g
Domain of g = Range of f

Ex. Show that the following are inverses of each other.

$$f(x) = 2x^3 - 1 \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

$$f(g(x)) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1 = x$$

Reflective Property of Inverse Functions

f contains (a, b) iff f^{-1} contains (b, a)

↓

"if and only if"

Existence of an Inverse Function

1. A function possesses an inverse iff it is 1 – 1.
2. If f is strictly monotonic on its entire domain, then it is 1 – 1 and hence, possesses an inverse.

Note: strictly monotonic means the function is increasing or decreasing over its entire domain.

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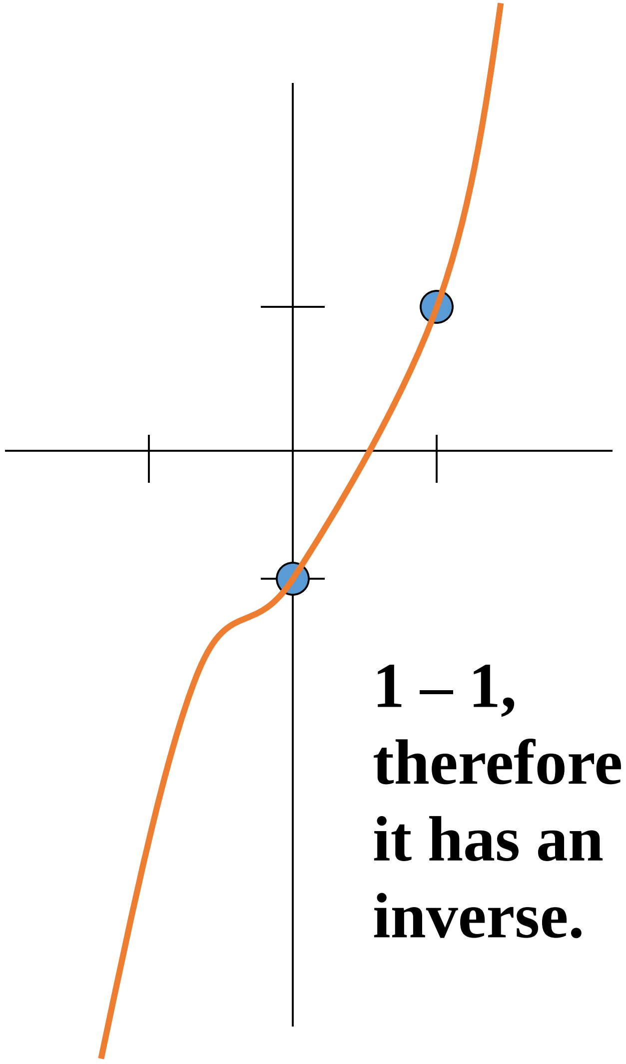
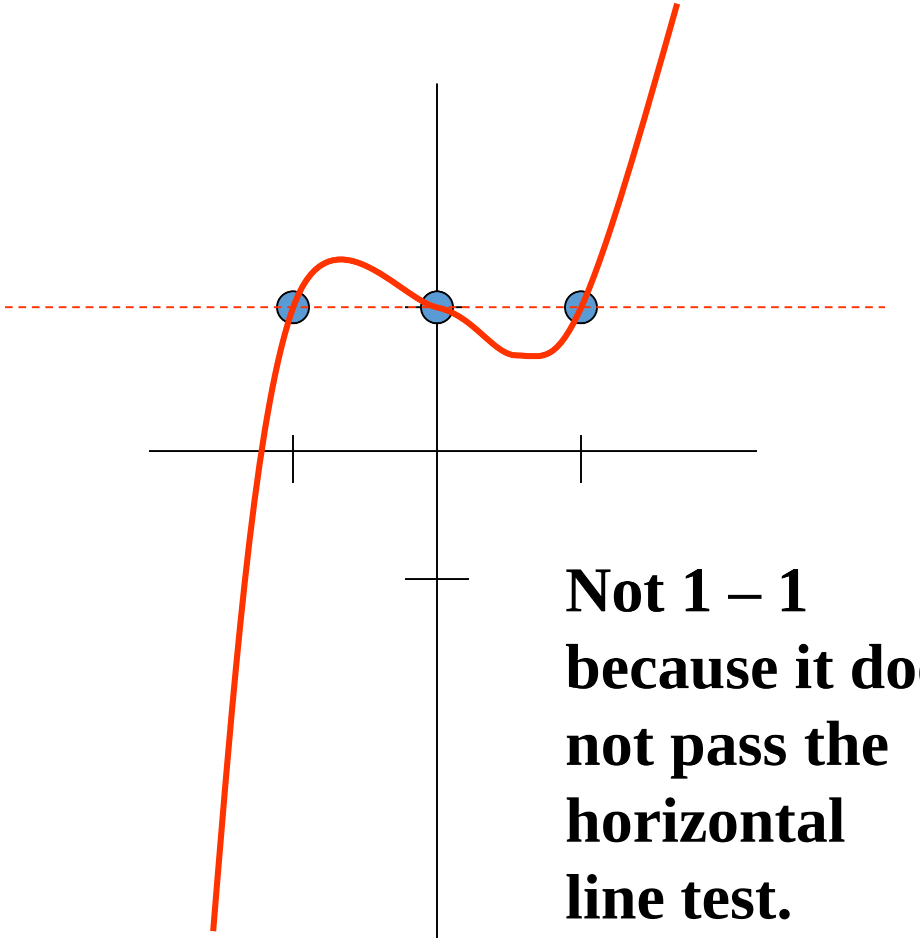
Let’s look at the following two functions.

a.) $f(x) = x^3 + x - 1$ and b.) $f(x) = x^3 - x + 1$



1 – 1, therefore it has an inverse.



Not 1 – 1 because it does not pass the horizontal line test.

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Find the inverse of $f(x)=\sqrt{2x-3}$

$$y=\sqrt{2x-3}$$

$$y^2=2x-3$$

$$y^2+3=2x$$

$$\frac{y^2+3}{2}=x$$

$$\frac{x^2+3}{2}=y$$

$$\frac{x^2+3}{2}=f^{-1}(x)$$

Domain of f(x)

$$\left[\frac{3}{2},\infty\right)$$

Range of f(x)

$$[0,\infty)$$

Domain of $f^{-1}(x)$ = Range of f(x)

and

Range of $f^{-1}(x)$ = Domain of f(x)

Steps for finding an inverse.

1. **solve for x**
2. **exchange x's and y's**
3. **replace y with f^{-1}**

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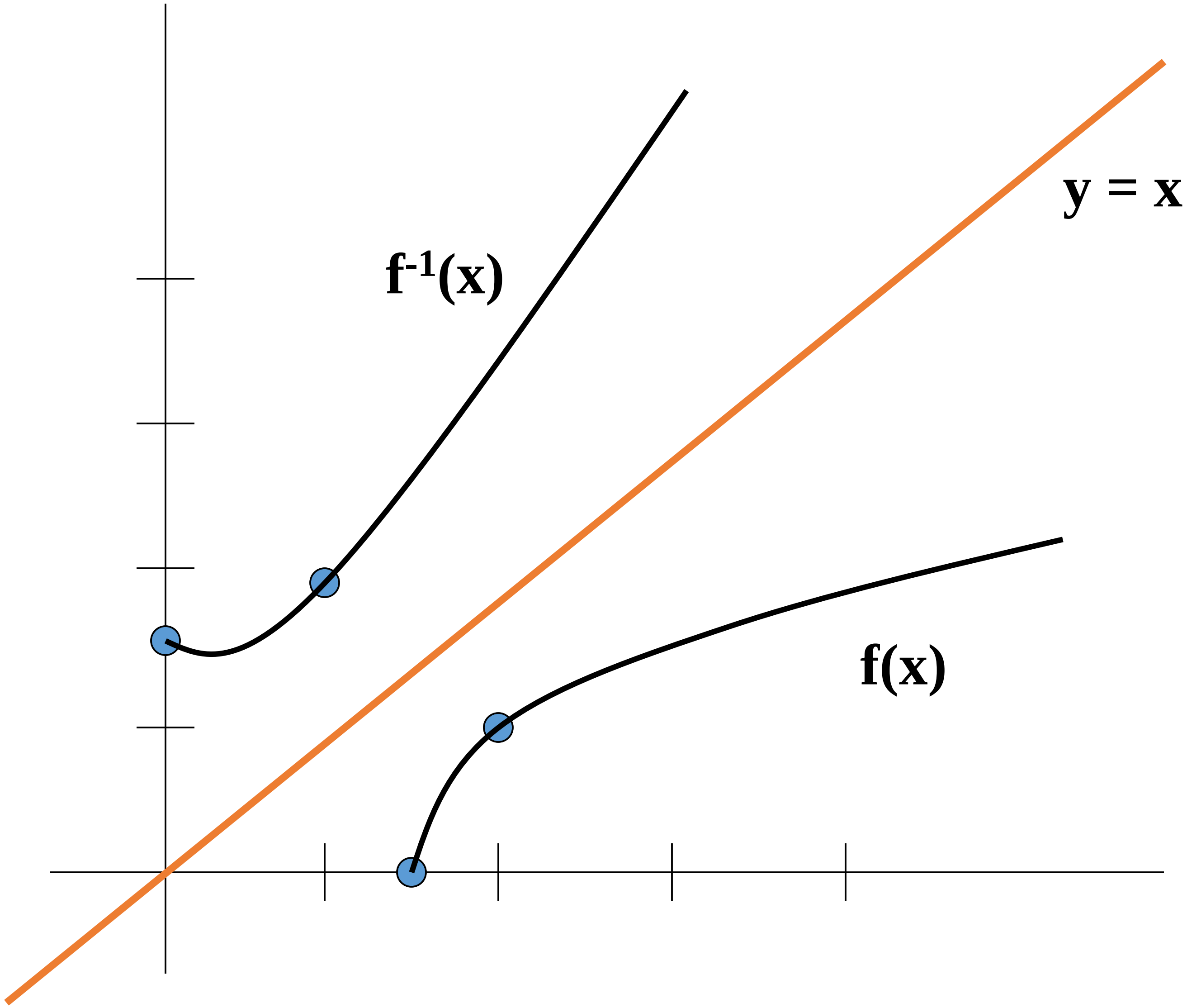

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The Derivative of an Inverse Function

If f is differentiable on its domain and possesses an inverse function g, then the derivative of g is given by

$$g'(x) = \frac{1}{f'(g(x))}$$

Graphs of inverse functions have reciprocal slopes.

Let f(x) = x^2 (for x >0) and let $f^{-1}(x) = \sqrt{x}$.

Show that the slopes of the graphs of f and f^{-1} are reciprocals at the following points. (2, 4) and (4, 2)

Find the derivatives of f and f^{-1}.

$$f'(x) = 2x \quad \text{and} \quad \left(f^{-1}\right)'(x) = \frac{1}{2\sqrt{x}}$$

At (2, 4), the slope of the graph of f is f'(2) = 4.
At (4, 2), the slope of the graph of f^{-1} is ¼.