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Complex Numbers

Definition of pure imaginary numbers:

Any positive real number b,

$$\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1} = bi$$

where i is the imaginary unit and bi is called the pure imaginary number.

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Definition of pure imaginary numbers:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

i is not a variable
it is a symbol for a specific number

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Simplify each expression.

1. $\sqrt{-81} = \sqrt{81}\cdot\sqrt{-1} = 9i$

2. $\sqrt{-121x^5} = \sqrt{121x^4}\cdot\sqrt{-1}\cdot\sqrt{x}$

$= 11x^2 i\cdot\sqrt{x}$

3. $\sqrt{-200x} = \sqrt{100}\cdot\sqrt{-1}\cdot\sqrt{2x}$

$= 10i\cdot\sqrt{2x}$

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Simplify each expression.

4. $8i \cdot 3i = 24i^2 = 24 \cdot -1$

Remember $i^2 = -1$ $\qquad = -24$

5. $\sqrt{-5} \cdot \sqrt{-20} = i\sqrt{5} \cdot i\sqrt{20}$

Remember that $\sqrt{-1} = i$

$= i^2 \cdot \sqrt{100} = -1 \cdot 10 = -10$

Remember $i^2 = -1$

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Cycle of "I"

$$i^0 = 1$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$

Simplify.

i^{12}

To figure out where we are in the cycle divide the exponent by 4 and look at the remainder.

$12 \div 4 = 3$ with remainder 0

So $i^{12} = i^{0} = 1$

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Simplify.

i^{17} **Divide the exponent by 4 and look at the remainder.**

$$17 \div 4 = 4 \text{ with remainder } 1$$

$$\text{So } i^{17} = i^{1} = i$$

Simplify.

$$i^{26}$$

Divide the exponent by 4 and look at the remainder.

$$26 \div 4 = 6 \text{ with remainder } 2$$

$$\text{So } i^{26} = i^{2} = -1$$

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Simplify.

i^{11} **Divide the exponent by 4 and look at the remainder.**

$11 \div 4 = 2$ with remainder 3

So $i^{11} = i^{3} = -i$

Definition of Complex Numbers

Any number in form $a+bi$, where a and b are real numbers and i is imaginary unit.

Definition of Equal Complex Numbers

Two complex numbers are equal if their real parts are equal and their imaginary parts are equal.

If $a + bi = c + di$,

then $a = c$ and $b = d$

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When adding or subtracting complex numbers, combine like terms.

$$Ex: \quad (8-3i)+(2+5i)$$

$$(8+2)+(-3i+5i)$$

$$10+2i$$

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Simplify.

$$(8+7i)+(-12+11i)$$

$$(8+-12)+(7i+11i)$$

$$-4+18i$$

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Simplify.

$$(9-6i)-(12+2i)$$

$$(9-12)+(-6i-2i)$$

$$-3-8i$$

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Multiplying complex numbers.

To multiply complex numbers, you use the same procedure as multiplying polynomials.

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Simplify.

$$(8+5i)(2-3i)$$

F O I L

$$16-24i+10i-15i^2$$

$$16-14i+15$$

$$31-14i$$

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Simplify.

$$(-6+2i)(5-3i)$$

F O I L

$$-30+18i+10i-6i^2$$

$$-30+28i+6$$

$$-24+28i$$