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Rational Expressions

Domain: the set of x-values.

The domain of a polynomial is always all Real numbers except for two cases:

1. Denominators
2. Radicals

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Determine the domain of each of the following:

a. $3x^3 + 4x + 7$

Is there a den. or radical in it?

$$No, \therefore \quad x \in \Re$$

is the domain.

b. $\sqrt{x-2}$

To find the domain with a radical, set the inside ≥ 0

$$x - 2 \geq 0$$

$$\therefore \; x \geq 2$$

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c. $$\frac{x+2}{x-3}$$

Since this example has a denominator in it, what x-value will make it undefined?

The domain will be all real numbers except x = 3

x - 3 = 0 or

x = 3

This is the answer

$$x \in \Re, x \neq 3$$

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Reduce the rational expression.

$$\frac{x^2+4x-12}{3x-6} = \frac{(x+6)\cancel{(x-2)}}{3\cancel{(x-2)}}$$

$$= \frac{x+6}{3}, x \neq 2$$

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a. $$\frac{x^3-4x}{x^2+x-2}=\frac{x(x-2)(x+2)}{(x+2)(x-1)}$$

$$=\frac{x(x-2)}{x-1},x\neq -2$$

b. $$\frac{12+x-x^2}{2x^2-9x+4}=\frac{(4-x)(3+x)}{(2x-1)(x-4)}=$$

$$\frac{-(x-4)(3+x)}{(2x-1)(x-4)}=-\frac{3+x}{2x-1},x\neq 4$$

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Multiply the Rational Expressions

$$\frac{2x^2+x-6}{x^2+4x-5}\bullet\frac{x^3-3x^2+2x}{4x^2-6x}=$$

$$\frac{\cancel{(2x-3)}(x+2)}{(x+5)\cancel{(x-1)}}\bullet\frac{\cancel{x}(x-2)\cancel{(x-1)}}{2\cancel{x}\cancel{(2x-3)}}=$$

$$\frac{(x+2)(x-2)}{2(x+5)},\ x\neq 0,1,\frac{3}{2}$$

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Divide the Rational Expressions

$$\frac{x^3-8}{x^2-4} \div \frac{x^2+2x+4}{x^3+8} = \frac{x^3-8}{x^2-4} \bullet \frac{x^3+8}{x^2+2x+4} =$$

Now factor everything.

$$\frac{\cancel{(x-2)}\cancel{(x^2+2x+4)}}{\cancel{(x+2)}\cancel{(x-2)}} \bullet \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x^2+2x+4}} =$$

$$= x^2-2x+4, \quad x \neq \pm 2$$

Subtract the Rational Expressions

$$\frac{x}{x-3}-\frac{2}{3x+4}=\frac{x(3x+4)-2(x-3)}{(x-3)(3x+4)}$$

$$=\frac{3x^2+4x-2x+6}{(x-3)(3x+4)}$$

$$=\frac{3x^2+2x+6}{(x-3)(3x+4)}$$

Subtract the Rational Expression

$$\frac{3}{x-1} - \frac{2}{x} + \frac{x+3}{x^2-1} =$$

(x-1)(x+1)

$$\frac{3x(x+1) - 2(x-1)(x+1) + (x+3)x}{x(x+1)(x-1)} =$$

$$\frac{3x^2 + 3x - 2x^2 + 2 + x^2 + 3x}{x(x-1)(x+1)} =$$

$$= \frac{2x^2+6x+2}{x(x-1)(x+1)} = \frac{2(x^2+3x+1)}{x(x+1)(x-1)}$$

Simplify the Compound Fractions

$$\frac{\left(\frac{2}{x}-3\right)}{\left(1-\frac{1}{x-1}\right)} = \frac{\frac{2-3x}{x}}{\frac{1(x-1)-1}{x-1}} =$$

x - 2

Now, invert the den. and mult.

$$= \frac{2-3x}{x} \bullet \frac{x-1}{x-2}, \quad x \neq 1$$

Simplifying a Compound Fraction by Multiplying by the LCD.

$$\frac{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)}{\left(\frac{1}{x} + \frac{1}{y}\right)} \bullet \frac{\frac{x^2y^2}{1}}{\frac{x^2y^2}{1}} = \frac{y^2 - x^2}{xy^2 + x^2y} = \frac{(y-x)\cancel{(y+x)}}{xy\cancel{(y+x)}}$$

Multiply both the top and bottom by x^2y^2