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Exponential & Logarithmic Functions

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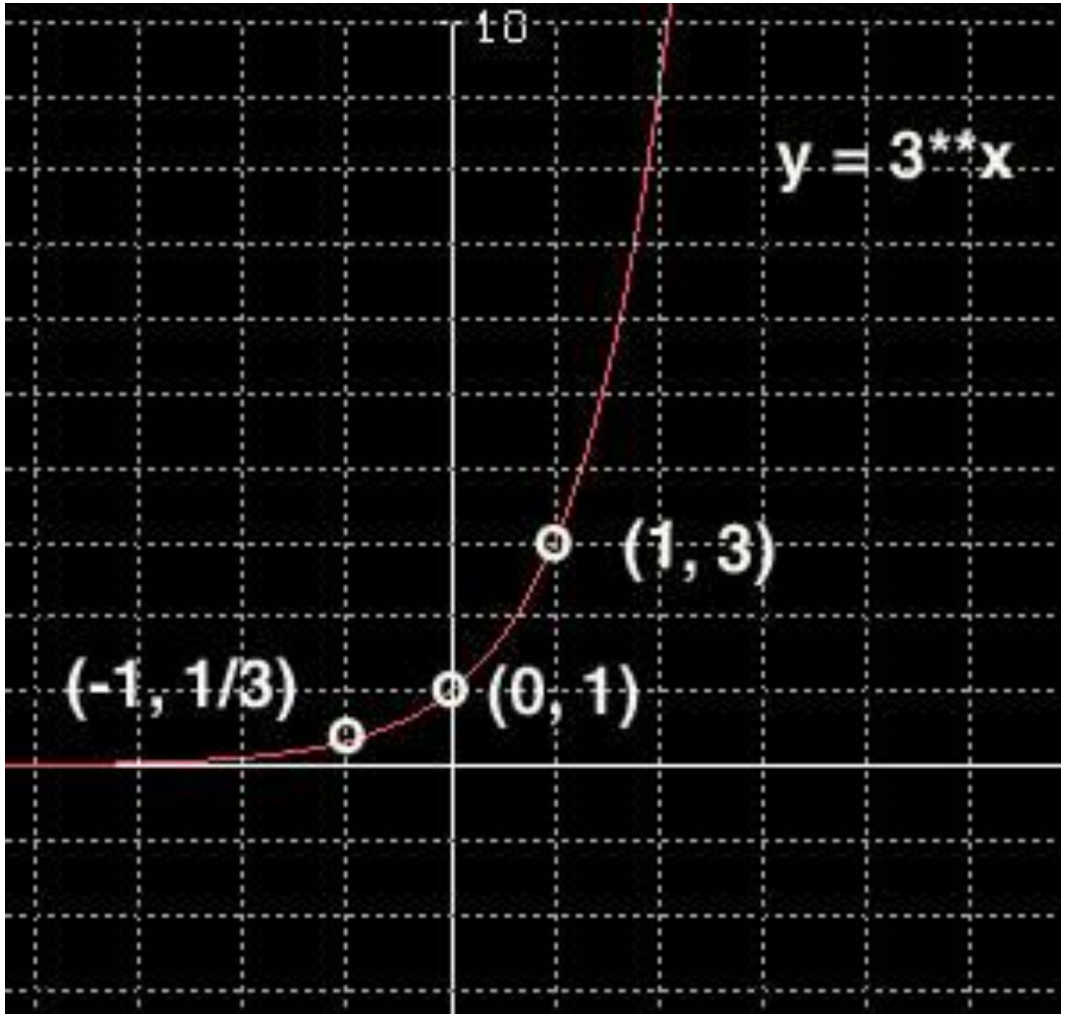
General Form of Exponential Function

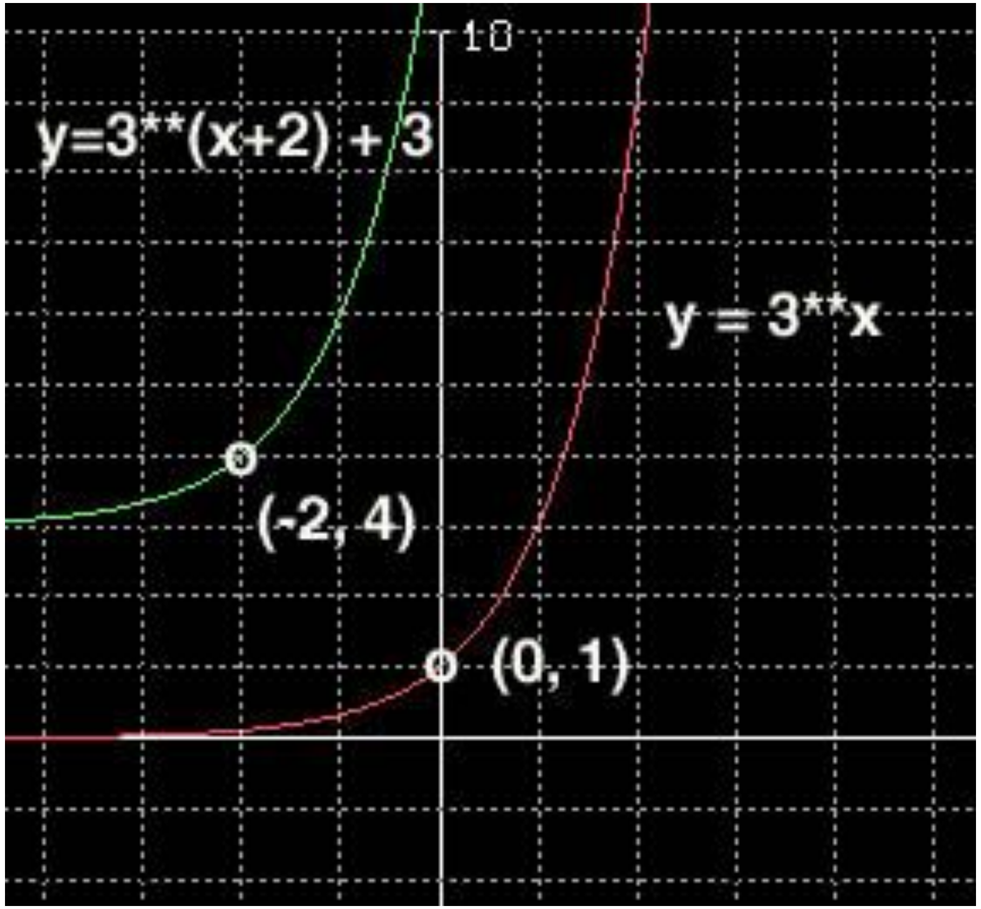
$y = b^x$ where $b > 1$

- Domain: All reals
- Range:

  $y > 0$
- x-intercept: None
- y-intercept: (0, 1)



General Form of Exponential Function $y = b^{(x + c)} + d$ where $b > 1$



- **c** moves graph left or right (opposite way)
- **d** move graph up or down (expected way)
- So $\mathbf{y=3^{(x+2)} + 3}$ moves the graph 2 units to the left and 3 units up
- (0, 1) to (– 2, 4)

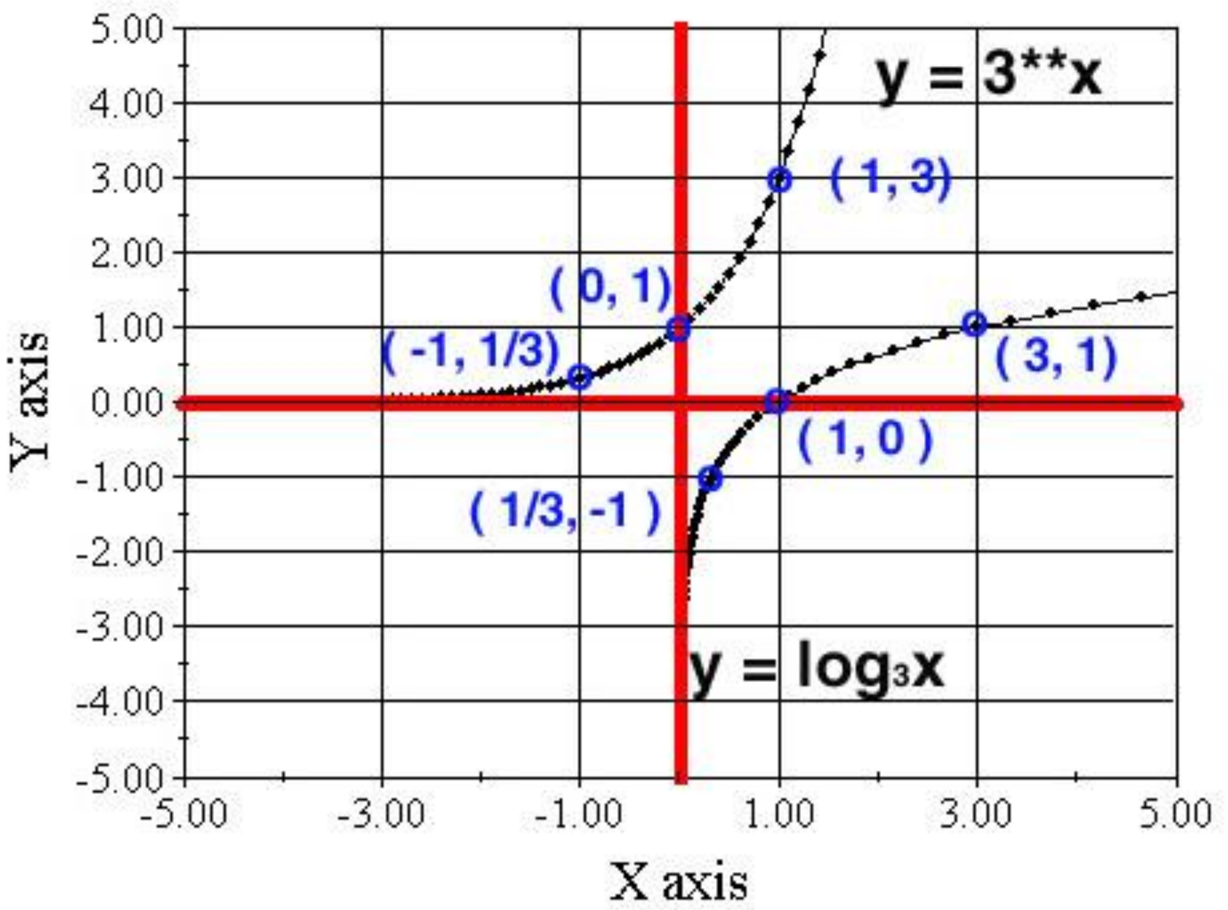
Relationships of Exponential ($y = b^x$) & Logarithmic ($y = \log_b x$) Functions

- $y = b^x$

- Domain: All Reals
- Range: $y > 0$
- x-intercept: None
- y-intercept: (0, 1)

- $y = \log_b x$ is the inverse of $y = b^x$
- Domain: $x > 0$
- Range: All Reals
- x-intercept: (1, 0)
- y-intercept: None

Relationships of Exponential ($y = b^x$) & Logarithmic ($y = \log_b x$) Functions



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Converting between Exponents & Logarithms

- $BASE^{EXPONENT}$ = POWER
- $4^2 = 16$
- 4 is the base. 2 is the exponent. 16 is the power.
- As a logarithm, $\log_{BASE}POWER=EXPONENT$
- $\log_4 16 = 2$

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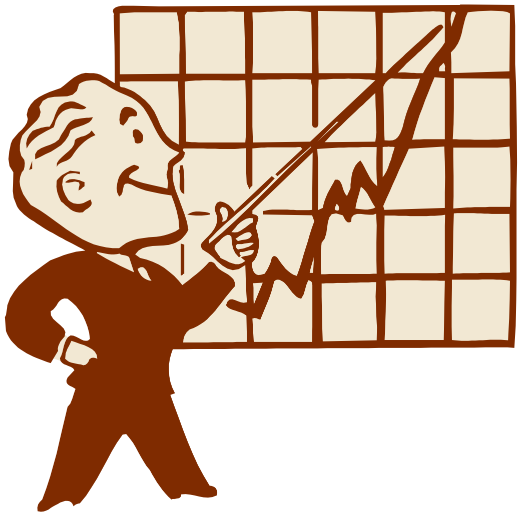
Logarithmic Abbreviations

- $\log_{10} x = \log x$ (Common log)
- $\log_e x = \ln x$ (Natural log)
- $e = 2.71828...$



Properties of Logarithms

- $\log_b(MN) = \log_b M + \log_b N$

  Ex: $\log_4(15) = \log_4 5 + \log_4 3$

- $\log_b(M/N) = \log_b M - \log_b N$

  Ex: $\log_3(50/2) = \log_3 50 - \log_3 2$

- $\log_b M^r = r \log_b M$

  Ex: $\log_7 10^3 = 3 \log_7 10$

- $\log_b(1/M) = \log_b M^{-1} = -1 \log_b M = -\log_b M$

  $\log_{11}(1/8) = \log_{11} 8^{-1} = -1 \log_{11} 8 = -\log_{11} 8$

Properties of Logarithms (Shortcuts)

- $\log_b 1 = 0$ **(because $b^0 = 1$)**
- $\log_b b = 1$ **(because $b^1 = b$)**
- $\log_b b^r = r$ **(because $b^r = b^r$)**

- $b^{\log_b M} = M$ **(because $\log_b M = \log_b M$)**

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Examples of Logarithms

- Simplify $\log 7 + \log 4 - \log 2 =$

$\log \frac{7*4}{2} = \log 14$

- Simplify $\ln e^2 =$

$2 \ln e = 2 \log_e e = 2 * 1 = 2$

- Simplify $e^{4 \ln 3 - 3 \ln 4} =$

$e^{\ln 3^4 - \ln 4^3} = e^{\ln 81/64} = e^{\log_e 81/64} = 81/64$

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Change-of-Base Formula

- $\log_b m = \dfrac{\log_a m}{\log_a b}$

- $\log_7 12 = \dfrac{\log 12}{\log 7}$

OR

- $\log_7 12 = \dfrac{\ln 12}{\ln 7}$

Exponential & Logarithmic Equations

- If $\log_b m = \log_b n$, then m = n.

  If $\log_6 2x = \log_6(x + 3)$,
  then 2x = x + 3 and x = 3.

- If $b^m = b^n$, then m = n.

  If $5^{1-x} = 5^{-2x}$, then 1 – x = – 2x and
  x = – 1.

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If your variable is in the exponent.....

- Isolate the base-exponent term.
- Write as a log. Solve for the variable.
- Example: $4^{x+3} = 7$
- $\log_4 7 = x + 3$ and $-3 + \log_4 7 = x$

  OR with change of bases:

  $$x = -3 + \frac{\log 7}{\log 4}$$

- Another method is to take the LOG of both sides.

Logarithmic Equations

- Isolate to a single log term.
- Convert to an exponent.
- Solve equation.

- Example: $\log x + \log(x - 15) = 2$
- $\log x(x - 15) = 2$ so $10^2 = x(x - 15)$ and
  $100 = x^2 - 15x$ and $0 = x^2 - 15x - 100$
  So $0 = (x - 20)(x + 5)$ so $x = 20$ or -5

That's All Folks !