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Sum and Difference Formulas

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Sum and Difference Formulas for Cosines

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

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(1) Find the exact value of $\cos(105^\circ)$

$$\cos(105^\circ) = \cos(60^\circ + 45^\circ) =$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ =$$

$$= \frac{1}{2} * \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} =$$

$$= \frac{\sqrt{2} - \sqrt{6}}{2}$$

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Theorem Sum and Difference Formulas for Sines

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

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(2) Find the exact value of $\sin(-\frac{\pi}{12})$

$$\sin(-\frac{\pi}{12}) = \sin(\frac{\pi}{4} - \frac{\pi}{3}) =$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3} =$$

$$= \frac{\sqrt{2}}{2} * \frac{1}{2} - \frac{\sqrt{2}}{2} * \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} =$$

$$= \frac{\sqrt{2} - \sqrt{6}}{2}$$

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It is known that $\sin\alpha = \frac{1}{2}, \pi/2 < \alpha < \pi$;

and $\cos\beta = \frac{-1}{3}, \pi/2 < \beta < \pi$, find the exact value of

(a) $\cos\alpha$ (b) $\sin\beta$ (c) $\cos(\alpha - \beta)$

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Theorem Sum and Difference Formulas for Tangents

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

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(9) Find the exact value $\tan(\sin^{-1}\frac{12}{13} - \cos^{-1}\frac{-3}{5})$

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Cofunction Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

(10)Establish the identity

- Cos $(3\pi/2 + \theta) = \sin \theta$