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THE DOT PRODUCT

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The definition of the product of two vectors is:

$$\mathbf{v} \cdot \mathbf{w} = a_1 a_2 + b_1 b_2$$

where $\mathbf{v} = < a_1, b_1 >$ and $\mathbf{w} = < a_2, b_2 >$

If $\mathbf{v} = < -2, 5 >$ and $\mathbf{w} = < 4, 1 >$, find $\mathbf{v} \cdot \mathbf{w}$

$$\mathbf{v} \cdot \mathbf{w} = (-2)4 + (5)1$$

$$= -8 + 5 = -3$$

This is called the dot product. Notice the answer is just a number NOT a vector.

What is the dot product of: <3, 4> and <-1, 2>

A. <2, 6>

B. <-3, 8>

C. -11

D. 5

What is the dot product of:
<-2, 6> and <8,5 >

A. 14

B. 24

C. 38

D. 46

Properties of the Dot Product

- Let **u**, **v**, and **w** be vectors in the plane or in space and let *c* be a scalar.

1. $u \bullet v = v \bullet u$
2. $0 \bullet v = 0$
3. $u \bullet (v + w) = u \bullet v + u \bullet w$
4. $v \bullet v = \|v\|^2$
5. $c(u \bullet v) = cu \bullet v = u \bullet cv$

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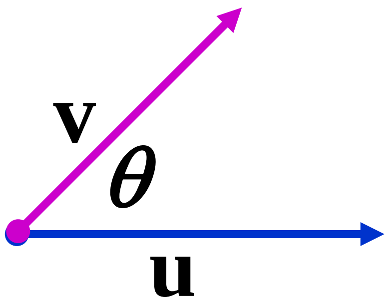
The dot product is useful for several things. One of the important uses is in a formula for finding the angle between two vectors that have the same initial point.

If $\mathbf{u}$ and $\mathbf{v}$ are two nonzero vectors, the angle θ, $0 \le \theta < \pi$, between $\mathbf{u}$ and $\mathbf{v}$ is determined by the formula

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$



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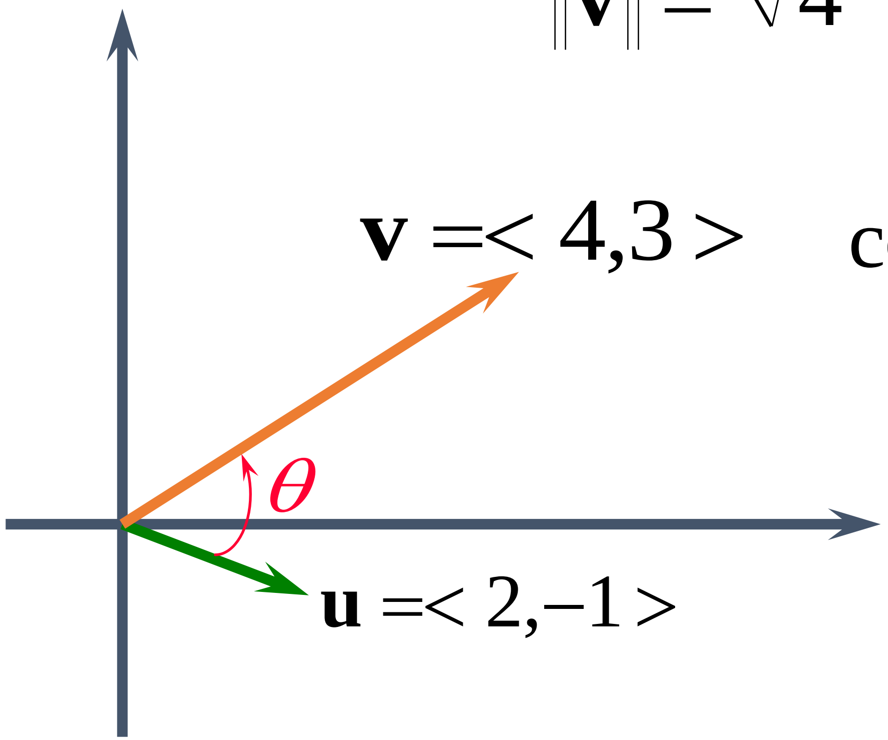
Find the angle θ between $\mathbf{u} = \langle 2,-1 \rangle$ and $\mathbf{v} = \langle 4,3 \rangle$.

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$\mathbf{u}\cdot\mathbf{v} = (2)4 + (-1)3 = 8 - 3 = 5$$

$$\|\mathbf{u}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$\|\mathbf{v}\| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$



$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{5}{\sqrt{5}\cdot 5} = \frac{1}{\sqrt{5}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{5}} \approx 63.4^\circ$$

What is the angle between the vectors **u** =<0, -5> and **v** = <1, -4>?

A. 60°

B. 14°

C. 40°

D. 17°

What is the angle between the vectors **u** =<-2, 3> and **v** = <-4, -2>?

A. 78°
B. 100°
C. 82.9°
D. 93°

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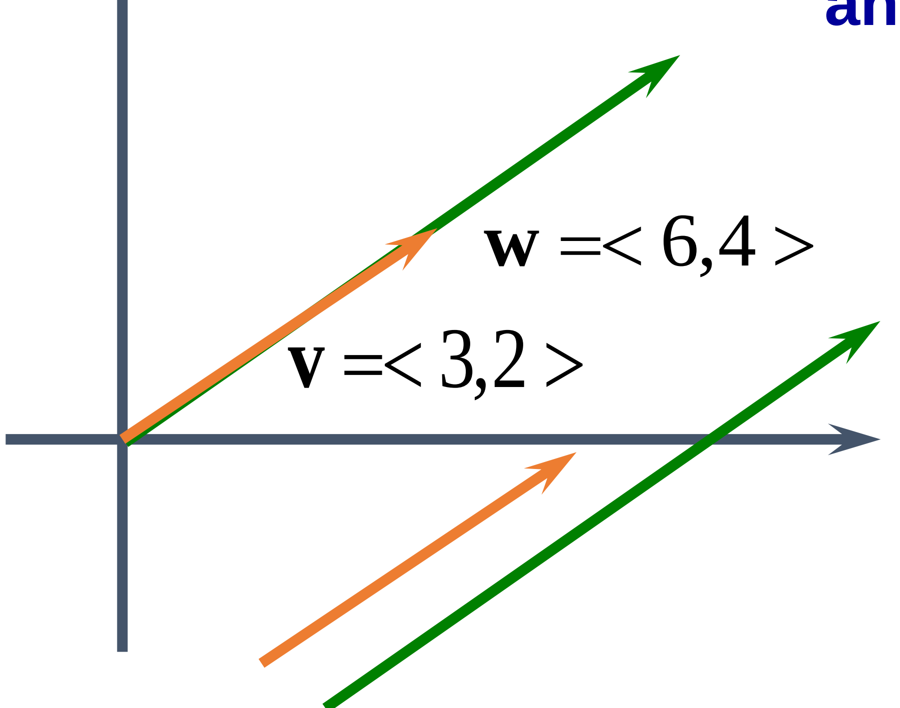
Find the angle between the vectors

v = <3, 2> and **w** = <6, 4>

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\,\|\mathbf{w}\|} = \frac{18+8}{\sqrt{13}\sqrt{52}} = \frac{26}{\sqrt{676}} = 1$$

$$\theta = \cos^{-1}1 = 0^\circ$$



What does it mean when the angle between the vectors is 0?

The vectors have the same direction. We say they are parallel because remember vectors can be moved around as long as you don't change magnitude or direction.

Orthogonal (Perpendicular) Vectors

- Two vectors are orthogonal if their dot product is 0

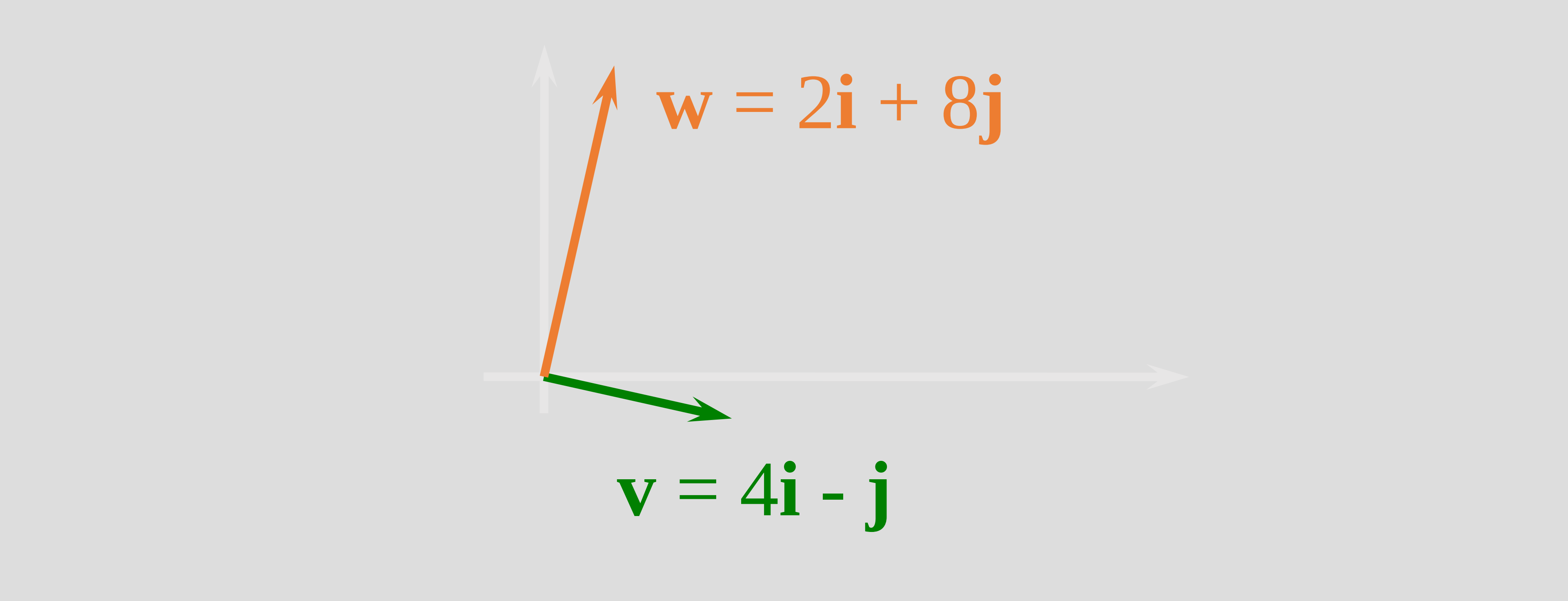
$$u \bullet v = 0$$

- Example:

$$\text{Let } u = \langle 2, -3 \rangle \text{ and } v = \langle 3, 2 \rangle$$

$$u \bullet v = (2)(3) + (-3)(2) = 0$$

so these vectors are othogonal



Determine whether the vectors **v** = 4**i** - **j** and **w** = 2**i** + 8**j** are orthogonal.

compute their dot product and see if it is 0

$$\mathbf{v} \cdot \mathbf{w} = (4)2 + (-1)8 = 0$$

$$\mathbf{v} \cdot \mathbf{w} = 0$$

The vectors v and w are orthogonal.

Are the vectors <2, -4> and <6, 3> orthogonal?

A. Yes

B. No

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Are the vectors <-5, -3> and <6, 10> orthogonal?

A. Yes

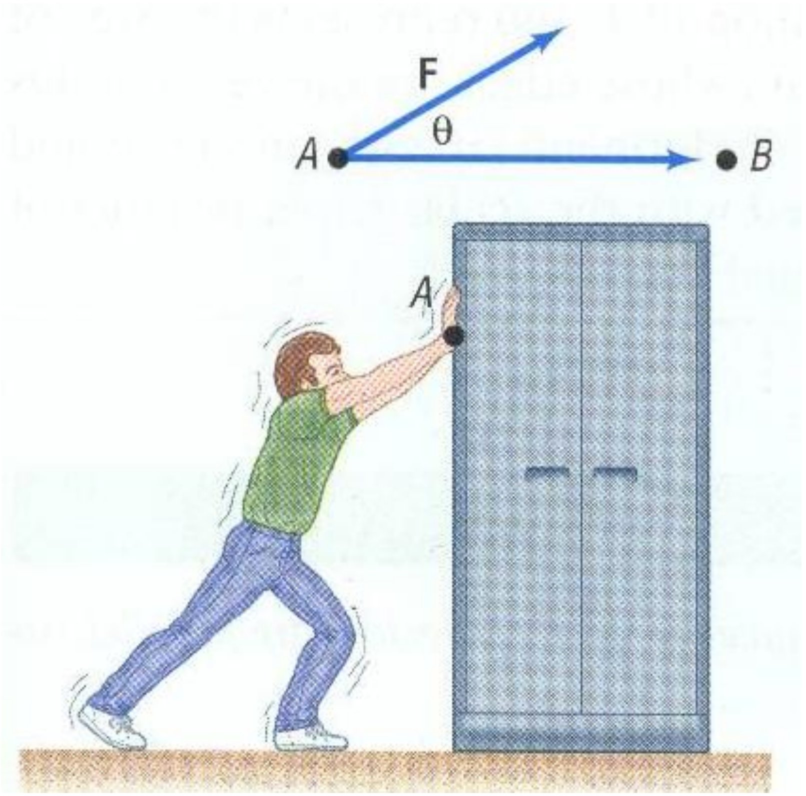
B. No

A use of the dot product is found in the formula below:

The work ***W*** done by a constant force **F** in moving an object from *A* to *B* is defined as

$$W = \mathbf{F} \cdot \overrightarrow{AB}$$

This means the force is in some direction given by the vector F but the line of motion of the object is along a vector from A to B



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Work Example

Constant force of 40 pounds in the direction of 25 degrees with the horizontal. The object is moved 20 feet, what is the work done?

Steps:

Find the x component of the force:

40cos(25) = 36.25

Multiply by the distance: 20 feet

36.35(20)=725