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The Law of Sines
The Law of Cosines

Review how we got the Law of Sines

Draw a large triangle, and label vertices A, B, and C. Be neat– make the sides as straight as you can.

Again neatly, sketch an altitude from vertex B, and label this altitude h.

What relationship is there between h and angle C? (You may want to consider a trig ratio.)

What is the area of a triangle? How could you write it with a trig ratio?

POD– a hands-on experience.

Area = ½ (base)(height).

In this triangle, area would be ½ ba(sinC).

But wait! Draw a height from vertex C. What happens then?

Draw a height from vertex A. What about that?

POD– a hands-on experience.

No matter how we orient the triangle, the area will always be ½ (base)(height). So

$$area = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

Moving on

Start with this.

$$\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

Multiply each term by 2 and divide each term by abc.

$$\frac{ab\sin C}{abc} = \frac{bc\sin A}{abc} = \frac{ac\sin B}{abc}$$

Simplify, what is the final result?

The Law of Sines

You've just built the Law of Sines.

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

or $$\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$$

What does this tell us?

It is true for all angles, not just acute ones.

The Law of Sines

You've just built the Law of Sines.

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

What does this tell us?

The ratio between the length of a side in a triangle, and the sine of the opposite angle is constant in that triangle.

Is this cool or what?

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Use it

You can use the Law of Sines to solve triangles when given AAS, ASA, or SSA. (What does that mean?) (What is the caution?)

Solve ΔABC given $\alpha = 48°$, $\gamma = 57°$, and $b = 47$. (What condition is this?)

Draw a diagram if it helps.

Use it

You can use the Law of Sines to solve triangles when given AAS, ASA, or SSA.

Solve ΔABC given $\alpha = 48°$, $\gamma = 57°$, and $b = 47$.

ASA: two angles and the side between

The third angle is a snap. Then use Law of Sines.

Use it

You can use the Law of Sines to solve triangles when given AAS, ASA, or SSA.

Solve ΔABC given $\alpha = 48°$, $\gamma = 57°$, and $b = 47$.

$\beta = 180° - 48° - 57°$

$$\frac{a}{\sin 48°} = \frac{47}{\sin 75°}$$

$$\frac{c}{\sin 57°} = \frac{47}{\sin 75°}$$

Use it

You can use the Law of Sines to solve triangles when given AAS, ASA, or SSA.

Solve ΔABC given $\alpha = 48°$, $\gamma = 57°$, and $b = 47$. (What condition is this?)

$\beta = 75°$ $a = 36$ $c = 41$

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Law of Cosines

We have three ways to write it. Here are two. What is the third?

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

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New

We have three ways to write it.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

What is the pattern?

What triangles would we use this tool for?

What happens if the angle is 90°?

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New

We have three ways to write it.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

What is the pattern?

What triangles would we use this tool for? SSS, SAS, SSA

What happens if the angle is 90°?

Use it

Work with an SSS condition.

If ΔABC has sides $a = 90$, $b = 70$, and $c = 40$, find the three angles.

Use it

Work with an SSS condition.

If ΔABC has sides a = 90, b = 70, and c = 40, find the three angles.

Start with the smallest angle (opposite which side?) to make sure to deal with an acute angle— no ambiguity if you use the Law of Sines later.

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Use it

Work with an SSS condition.

If ΔABC has sides a = 90, b = 70, and c = 40, find the three angles.

$$40^2 = 90^2 + 70^2 - 2 \cdot 90 \cdot 70 \cos C$$
$$1600 = 8100 + 4900 - 12600 \cos C$$
$$-11400 = -12600 \cos C$$
$$\cos C = .9048$$
$$C = 25.2°$$

Next, find the middle angle, since it has to be acute as well.

Use it

Work with an SSS condition.

If ΔABC has sides a = 90, b = 70, and c = 40, find the three angles.
You could also use the Law of Sines.

$$70^2 = 90^2 + 40^2 - 2 \cdot 90 \cdot 40 \cos B$$

$$4900 = 8100 + 1600 - 7200 \cos B$$

$$-4800 = -7200 \cos B$$

$$\cos B = .6667$$

$$B = 48.2°$$

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Use it

Work with an SSS condition.

If ΔABC has sides a = 90, b = 70, and c = 40, find the three angles.

$$B = 48.2^\circ$$

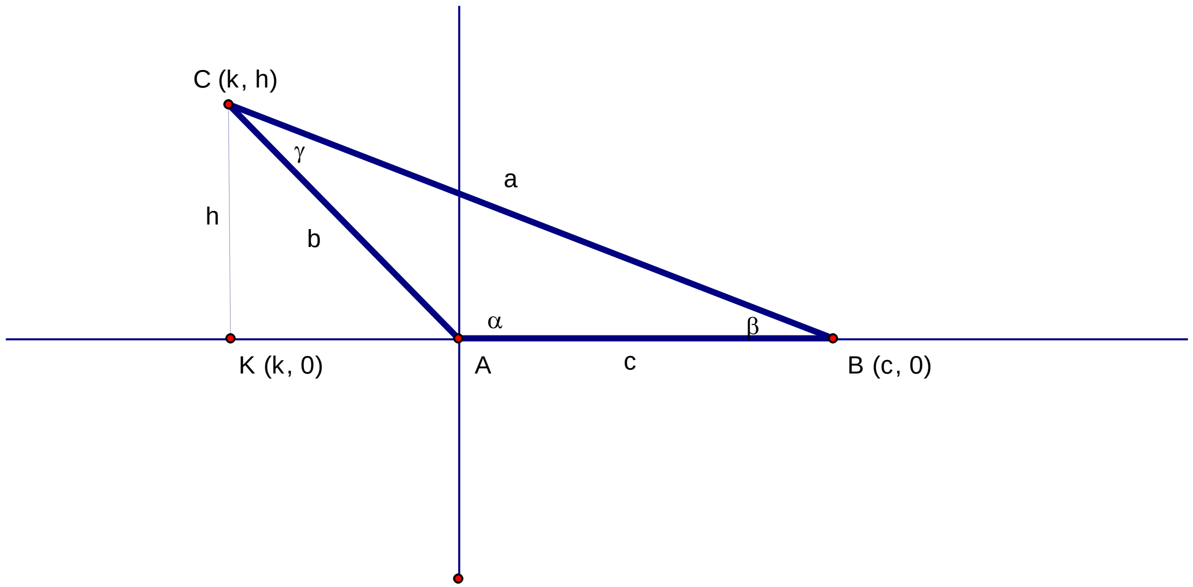
$$C = 25.2^\circ$$

$$A = 180^\circ - 25.2^\circ - 48.2^\circ = 106.6^\circ$$

How could you check your answer?

The Proof

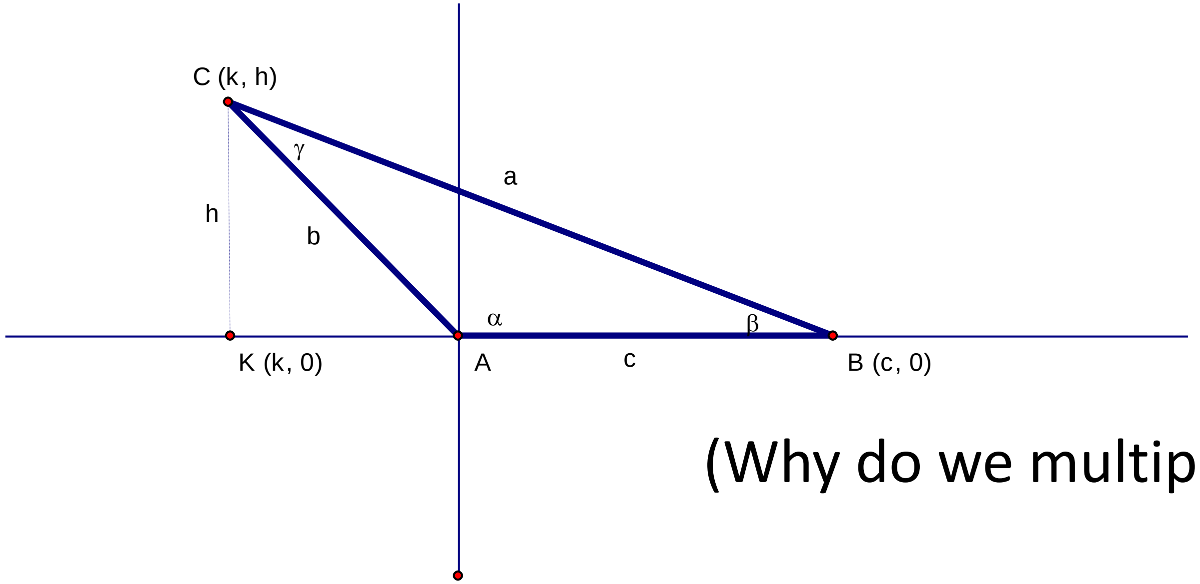
Let's start by looking at an obtuse triangle in standard position. What are h and k?



The Proof

Let's start by looking at an obtuse triangle in standard position.

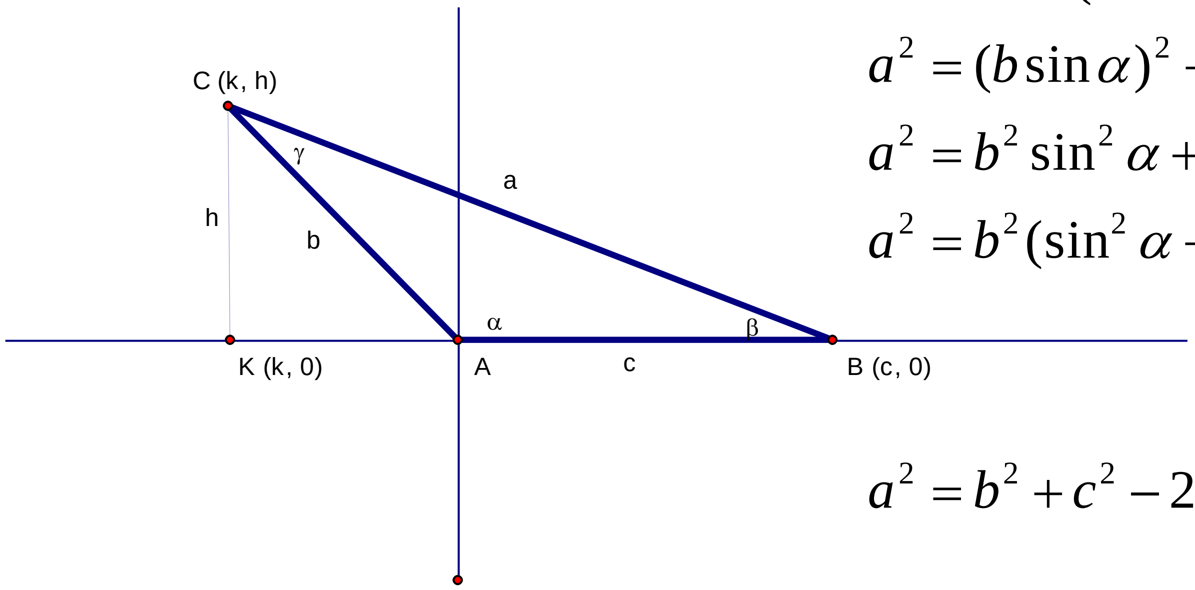
$k = b\cos\alpha \qquad h = b\sin\alpha$



(Why do we multiply by b?)

The Proof

Now for the algebra. We'll look at right ΔKBC.



$$k = b\cos\alpha$$

$$h = b\sin\alpha$$

$$a^2 = h^2 + (c-k)^2 = h^2 + c^2 - 2ck + k^2$$

$$a^2 = (b\sin\alpha)^2 + c^2 - 2c(b\cos\alpha) + (b\cos\alpha)^2$$

$$a^2 = b^2\sin^2\alpha + c^2 - 2bc\cos\alpha + b^2\cos^2\alpha$$

$$a^2 = b^2(\sin^2\alpha + \cos^2\alpha) + c^2 - 2bc\cos\alpha$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$