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Using Fundamental Trig Identities

Verifying Identities

And

Solving Trig Equations

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Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \csc u = \frac{1}{\sin u}$$

$$\cos u = \frac{1}{\sec u} \qquad \sec u = \frac{1}{\cos u}$$

$$\tan u = \frac{1}{\cot u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

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Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

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Cofunction Identities

Complimentary Angles

$$\sin\left(\frac{\pi}{2}-u\right) = \cos(u) \qquad \cos\left(\frac{\pi}{2}-u\right) = \sin(u)$$

$$\tan\left(\frac{\pi}{2}-u\right) = \cot(u) \qquad \cot\left(\frac{\pi}{2}-u\right) = \tan(u)$$

$$\sec\left(\frac{\pi}{2}-u\right) = \csc(u) \qquad \csc\left(\frac{\pi}{2}-u\right) = \sec(u)$$

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Even/Odd Identities

EVEN	ODD
$f(-x) = f(x)$	$f(-x) = -f(x)$
$\cos(-u) = \cos(u)$	$\sin(-u) = -\sin(u)$
$\sec(-u) = \sec(u)$	$\tan(-u) = -\tan(u)$
	$\cot(-u) = -\cot(u)$
	$\csc(-u) = -\csc(u)$

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Use the given to evaluate all six trig functions

$$\cot(\theta) = -5 \;\; and \;\; \sin(\theta) = \frac{\sqrt{26}}{26}$$

First determine that quadrant the given information holds true…….

What quadrant is tangent negative and sine positive??? II

$$if \;\; \cot(\theta) = -5 \;\; then \;\; \tan(\theta) = \frac{-1}{5}$$

$$if \;\; \sin(\theta) = \frac{\sqrt{26}}{26} \;\; then \;\; \csc(\theta) = \sqrt{26}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\left(\frac{\sqrt{26}}{26}\right)^2 + \cos^2(\theta) = 1$$

$$\frac{1}{26} + \cos^2(\theta) = 1$$

$$\cos^2(\theta) = \frac{25}{26}$$

$$\cos(\theta) = -\frac{5}{\sqrt{26}} = -\frac{5\sqrt{26}}{26}$$

$$if \;\; \cos(\theta) = \frac{-5}{\sqrt{26}}$$

$$then \;\; \sec(\theta) = \frac{-\sqrt{26}}{5}$$

Simplify a Trig Expression

$$\sin x \cos^2 x - \sin x$$

$$\sin x \left(\cos^2 x - 1\right)$$

$$\textit{remember}: \quad \sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - 1 = -\sin^2 x$$

$$\sin x \left(-\sin^2 x\right)$$

$$-\sin^3 x$$

Verify a Trig Identity

$$\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}=\csc\theta$$

$$\frac{\sin\theta\sin\theta+\cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta}=\csc\theta$$

$$\frac{\sin^2\theta+\cos\theta+\cos^2\theta}{(1+\cos\theta)\sin\theta}=\csc\theta$$

$$\frac{1+\cos\theta}{(1+\cos\theta)\sin\theta}=\csc\theta$$

$$\frac{1}{\sin\theta}=\csc\theta$$

$$\csc\theta=\csc\theta$$

Work on one side only!

Work DOWN the page, not across!

Verify a Trig Identity

Use the table feature and graphing utility to check your result.

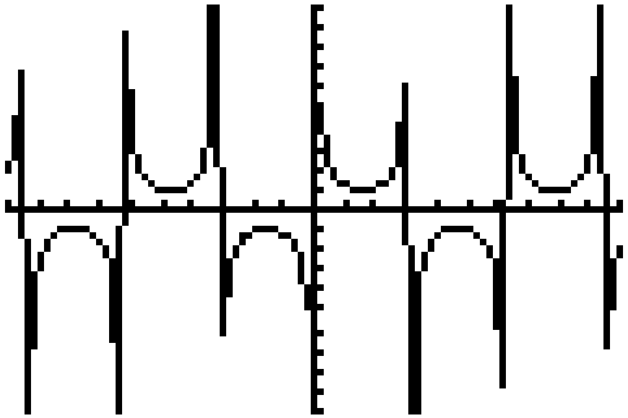
$$\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}=\csc\theta$$

Select the path style for y2 so you can see the tracing

$$y1=\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta} \qquad y2=\csc\theta$$

X	Y1	Y2
-3.142	ERROR	ERROR
-2.749	-2.613	-2.613
-2.356	-1.414	-1.414
-1.963	-1.082	-1.082
-1.571	-1	-1
-1.178	-1.082	-1.082
-.7854	-1.414	-1.414

X=-.785398163397



Verify a Trig Identity

$$\frac{\sin\theta+\cos\theta}{\sin\theta} - \frac{\cos\theta-\sin\theta}{\cos\theta} = \sec\theta\csc\theta$$

$$\frac{\cos\theta(\sin\theta+\cos\theta) - \sin\theta(\cos\theta-\sin\theta)}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{\cos\theta\sin\theta+\cos^2\theta - \sin\theta\cos\theta+\sin^2\theta}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\sin\theta}\bullet\frac{1}{\cos\theta} = \sec\theta\csc\theta$$

$$\csc\theta\sec\theta = \sec\theta\csc\theta$$

Factoring Trig Expressions

$$\sin^2 x - 1 \qquad a^2 - 1$$

$$(\sin x - 1)(\sin x + 1) \qquad (a-1)(a+1)$$

$$4\tan^2\theta + \tan\theta - 3 \qquad 4a^2 + a - 3$$

$$(4\tan\theta - 3)(\tan\theta + 1) \qquad (4a-3)(a+1)$$

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Factoring Trig Expressions

$$2\cos^2\alpha - \cos\alpha - 6 \qquad 2a^2 - a - 6$$

$$(2\cos\alpha + 3)(\cos\alpha - 2) \qquad (2a+3)(a-2)$$

$$\sec^3\theta - 1 \qquad a^3 - 1$$

$$(\sec\theta - 1)(\sec^2\theta + \sec\theta + 1) \qquad (a-1)(a^2+a+1)$$

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Factoring Trig Expressions

$$\sec^2\alpha - \tan\alpha - 3$$

$$remember:\ 1+\tan^2 x = \sec^2 x$$

$$\left(1+\tan^2\alpha\right) - \tan\alpha - 3$$

$$\tan^2\alpha - \tan\alpha - 2 \qquad a^2-a-2$$

$$(\tan\alpha - 2)(\tan\alpha + 1) \qquad (a-2)(a+1)$$

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Factoring Trig Expressions

$$\csc^4 x - \cot^4 x$$

$$\left(\csc^2 x - \cot^2 x\right)\left(\csc^2 x + \cot^2 x\right)$$

$$\textit{remember}: \ 1 + \cot^2 x = \csc^2 x$$

$$1 = \csc^2 x - \cot^2 x$$

$$1\left(\csc^2 x + \cot^2 x\right)$$

$$\csc^2 x + \cot^2 x$$

$$1 + \cot^2 x + \cot^2 x$$

$$1 + 2\cot^2 x$$

Factoring Trig Expressions

$$\cos^3 x - \cos^2 x - \cos x + 1$$

$$\cos^2 x(\cos x - 1) - 1(\cos x - 1)$$

$$(\cos x - 1)(\cos^2 x - 1)$$

$$(\cos x - 1)(\cos x - 1)(\cos x + 1)$$

$$(\cos x - 1)^2(\cos x + 1)$$

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Add & Subtract Trig Expressions

$$\frac{1}{1+\sin x} - \frac{1}{1-\sin x}$$

$$\frac{1(1-\sin x) - 1(1+\sin x)}{(1+\sin x)(1-\sin x)}$$

$$\frac{1-\sin x - 1-\sin x}{1-\sin^2 x}$$

$$\frac{-2\sin x}{\cos^2 x}$$

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Add & Subtract Trig Expressions

$$\frac{\sin x}{1-\cos x}+\frac{1+\cos x}{\sin x}$$

$$\frac{\sin^2 x+(1+\cos x)(1-\cos x)}{(1-\cos x)\sin x}$$

$$\frac{1-\cos^2 x+1-\cos^2 x}{(1-\cos x)\sin x}$$

$$\frac{2-2\cos^2 x}{(1-\cos x)\sin x}$$

$$\frac{2(1-\cos^2 x)}{(1-\cos x)\sin x}$$

$$\frac{2\cancel{(1-\cos x)}(1+\cos x)}{\cancel{(1-\cos x)}\sin x}$$

$$\frac{2(1+\cos x)}{\sin x}$$

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Add & Subtract Trig Expressions

$$\cot x - \frac{\csc^2 x}{\cot x}$$

$$\frac{\cot^2 x - \csc^2 x}{\cot x}$$

$$\frac{\csc^2 x - 1 - \csc^2 x}{\cot x}$$

$$\frac{-1}{\cot x}$$

$$-\tan x$$

Verify Trig Identities

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Verify Trig Identities - Guidelines

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants with tangents, and cosecants with cotangents.
4. If the preceding guidelines do not help, try converting all terms to sines and cosines.
5. Always try something. Even making an attempt that leads to a dead end provides insight.

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Verify Trig Identities

$$\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$$

$$1 - \frac{1}{\sec^2 x} = \sin^2 x$$

$$1 - \cos^2 x = \sin^2 x$$

$$\sin^2 x = \sin^2 x$$

Verify Trig Identities

$$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=2\csc^2\theta$$

$$\frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}=2\csc^2\theta$$

$$\frac{2}{1-\cos^2\theta}=2\csc^2\theta$$

$$\frac{2}{\sin^2 x}=2\csc^2\theta$$

$$2\square\frac{1}{\sin^2 x}=2\csc^2\theta$$

$$2\csc^2\theta = 2\csc^2\theta$$

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Verify Trig Identities

$$\tan\theta + \cot\theta = \sec\theta \cdot \csc\theta$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta \cdot \csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta} = \sec\theta \cdot \csc\theta$$

$$\frac{1}{\cos\theta \cdot \sin\theta} = \sec\theta \cdot \csc\theta$$

$$\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \cdot \csc\theta$$

$$\sec\theta \cdot \csc\theta = \sec\theta \cdot \csc\theta$$

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Verify Trig Identities

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$$

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$\sec\theta + \tan\theta = \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$\sec\theta + \tan\theta = \frac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$\sec\theta + \tan\theta = \frac{(1+\sin\theta)}{\cos\theta}$$

$$\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta + \tan\theta = \sec\theta + \tan\theta$$

Verify Trig Identities

$$\frac{1-\cos x}{\cos x} = \frac{\tan^2 x}{1+\sec x}$$

$$\frac{1-\cos x}{\cos x} = \frac{\sec^2 x - 1}{1+\sec x}$$

$$\frac{1-\cos x}{\cos x} = \frac{(\sec x - 1)(\sec x + 1)}{1+\sec x}$$

$$\frac{1-\cos x}{\cos x} = \sec x - 1$$

$$\frac{1-\cos x}{\cos x} = \frac{1}{\cos x} - 1$$

$$\frac{1-\cos x}{\cos x} = \frac{1-\cos x}{\cos x}$$

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Verify Trig Identities

$$\sec^2 x - \cot^2\left(\tfrac{\pi}{2} - x\right) = 1$$

$$\sec^2 x - \tan^2(x) = 1$$

$$1 + \tan^2(x) - \tan^2(x) = 1$$

$$1 = 1$$

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Verify Trig Identities

$$\sqrt{\frac{1-\sin x}{1+\sin x}} = \frac{1-\sin x}{|\cos x|}$$

$$\sqrt{\frac{(1-\sin x)}{(1+\sin x)} \cdot \frac{(1-\sin x)}{(1-\sin x)}}$$

$$\sqrt{\frac{(1-\sin x)^2}{(1-\sin^2 x)}}$$

$$\sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$$

$$\frac{1-\sin x}{|\cos x|} = \frac{1-\sin x}{|\cos x|}$$

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Verify Trig Identities

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cdot \cos x$$

$$\frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$$

$$\sin^2 x - \sin x \cos x + \cos^2 x$$

$$1 - \sin x \cos x = 1 - \sin x \cos x$$

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Verify Trig Identities

$$\cos x - 2\cos x \sin^2 x + \cos x \sin^4 x = \cos^5 x$$

$$\cos x\left(1 - 2\sin^2 x + \sin^4 x\right) = \cos^5 x$$

$$\cos x\left(1 - \sin^2 x\right)^2 = \cos^5 x$$

$$\cos x\left(\cos^2 x\right)^2 = \cos^5 x$$

$$\cos x\left(\cos^4 x\right) = \cos^5 x$$

$$\cos^5 x = \cos^5 x$$

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Verify Trig Identities

$$\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$$

$$\left(\sin^2 x\right)^2 + \cos^4 x$$

$$\left(1 - \cos^2 x\right)^2 + \cos^4 x$$

$$1 - 2\cos^2 x + \cos^4 x + \cos^4 x$$

$$1 - 2\cos^2 x + 2\cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$$

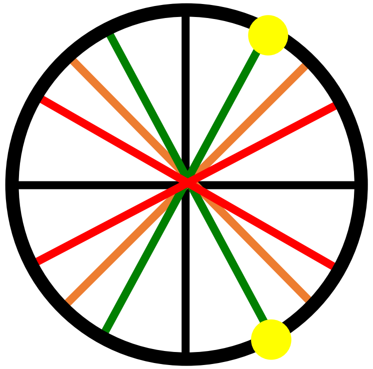
Solving Trig Equations

Solving Trig Equations

$$2\cos x - 1 = 0$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$



$$on \ \left[0, 2\pi\right)$$

$$x = \frac{\pi}{3} \quad or \quad x = \frac{5\pi}{3}$$

$$on \ \left(-\infty, +\infty\right)$$

$$x = \frac{\pi}{3} + 2n\pi \quad or \quad x = \frac{5\pi}{3} + 2n\pi$$

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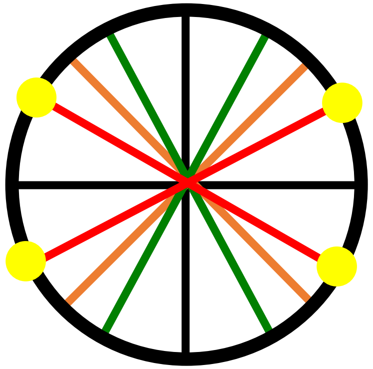
Solving Trig Equations

$$4\tan^2 x - 1 = \tan^2 x$$

$$3\tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm\frac{\sqrt{3}}{3}$$



$$on \ \left[0, 2\pi\right)$$

$$x = \frac{\pi}{6} \ or \ x = \frac{7\pi}{6}$$

$$or \ x = \frac{5\pi}{6} \ or \ x = \frac{11\pi}{6}$$

$$on \ \left(-\infty, +\infty\right)$$

$$x = \frac{\pi}{6} + n\pi \quad or \quad x = \frac{5\pi}{6} + n\pi$$

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Solving Trig Equations

$$\sec x - 2\tan x = 0$$

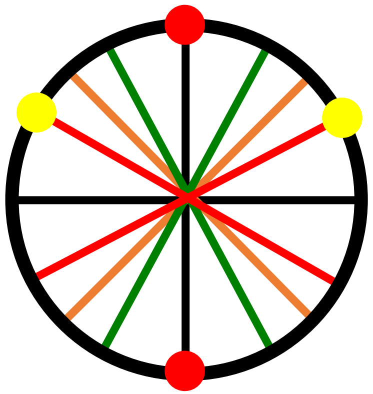
$$\frac{1}{\cos x} - \frac{2\sin x}{\cos x} = 0$$

$$\frac{1-2\sin x}{\cos x} = 0$$

$$\cos x \neq 0$$

$$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x\left(\frac{1-2\sin x}{\cos x}\right) = 0 \cdot \cos x$$



$$1 - 2\sin x = 0$$

$$\sin x = \frac{1}{2}$$

$$on \ \ (-\infty, +\infty)$$

$$x = \frac{\pi}{6} + 2n\pi \quad or \quad x = \frac{5\pi}{6} + 2n\pi$$

Solving Trig Equations

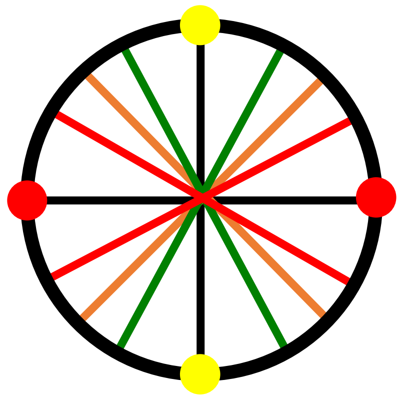
$$\cos x - \cot x = 0$$

$$\cos x - \frac{\cos x}{\sin x} = 0$$

$$\cos x\left(1 - \frac{1}{\sin x}\right) = 0$$

$$\sin x \neq 0$$

$$x \neq 0, \pi$$



$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 - \frac{1}{\sin x} = 0$$

$$1 = \frac{1}{\sin x}$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$on \ \ (-\infty, +\infty)$$

$$x = \frac{\pi}{2} + n\pi$$

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Solving Trig Equations

$$2\sin^2 x - 5\sin x = 3$$

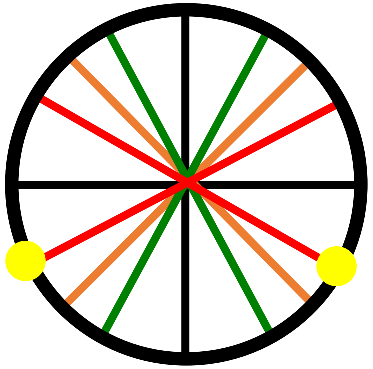
$$2\sin^2 x - 5\sin x - 3 = 0$$

$$(2\sin x + 1)(\sin x - 3) = 0$$

$$2\sin x + 1 = 0 \qquad \sin x - 3 = 0$$

$$2\sin x = -1 \qquad \sin x = 3$$

$$\sin x = -\tfrac{1}{2} \qquad \varnothing$$



$$x = \tfrac{7\pi}{6} \text{ or } x = \tfrac{11\pi}{6}$$

$$on \ (-\infty, +\infty)$$

$$x = \tfrac{7\pi}{6} + 2n\pi \ \text{ or } \ x = \tfrac{11\pi}{6} + 2n\pi$$

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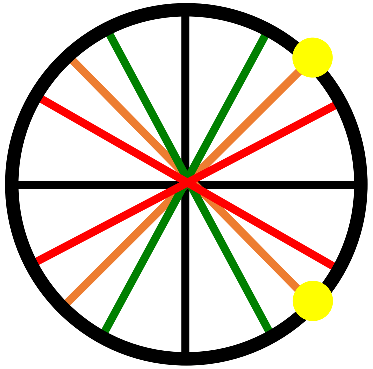
Solving Trig Equations

$$2\cos 2x - \sqrt{2} = 0$$

$$2\cos 2x = \sqrt{2}$$

$$\cos 2x = \frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4} + 2n\pi \quad or \quad 2x = \frac{7\pi}{4} + 2n\pi$$



$$x = \frac{\pi}{8} + n\pi \quad or \quad x = \frac{7\pi}{8} + n\pi$$

$$on \ \left[0, 2\pi\right)$$

$$\left\{\frac{\pi}{8}, \frac{9\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}\right\}$$

Sum & Difference Formulas

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Sum & Difference Formulas

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

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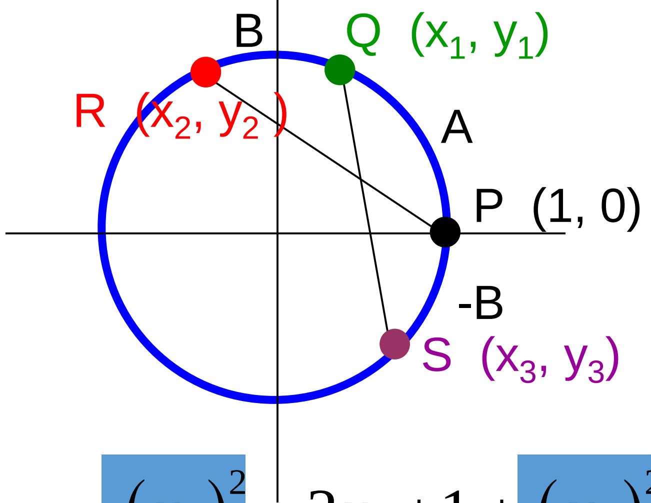
Proof of cos(A+B) = cosA•cosB - sinA•sinB



R (x_2, y_2) → (cos(A+B), sin(A+B))

Q (x_1, y_1) → (cosA, sinA)

S (x_3, y_3) → (cos(-B), sin(-B))

$$\overline{PR} = \overline{SQ}$$

$$\sqrt{[x_2-1]^2+[y_2-0]^2} = \sqrt{[x_1-x_3]^2+[y_1-y_3]^2}$$

$$[x_2-1]^2+[y_2-0]^2 = [x_1-x_3]^2+[y_1-y_3]^2$$

$$(x_2)^2-2x_2+1+(y_2)^2 = (x_1)^2-2x_1x_3+(x_3)^2+(y_1)^2-2y_1y_3+(y_3)^2$$

$$\left[(x_2)^2+(y_2)^2\right]-2x_2+1 = \left[(x_1)^2+(y_1)^2\right]+\left[(x_3)^2+(y_3)^2\right]-2x_1x_3-2y_1y_3$$

$$1-2x_2+1 = 1+1-2x_1x_3-2y_1y_3$$

$$-2x_2 = -2x_1x_3-2y_1y_3$$

$$x_2 = x_1x_3+y_1y_3$$

$$\cos(A+B) = \cos A\cos(-B)+\sin A\sin(-B)$$

$$\cos(A+B) = \cos A\cos B-\sin A\sin B$$

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Proof of
sin(A+B) = sinA•cosB + cosA•sinB

Note: This proof uses the cofunction identities.

$$\sin(A+B) = \cos\left[\tfrac{\pi}{2} - (A+B)\right]$$
$$= \cos\left[\tfrac{\pi}{2} - A - B\right]$$
$$= \cos\left[\left(\tfrac{\pi}{2} - A\right) - B\right]$$
$$= \cos\left(\tfrac{\pi}{2} - A\right)\cdot\cos B + \sin\left(\tfrac{\pi}{2} - A\right)\cdot\sin B$$
$$= \sin A\cdot\cos B + \cos A\cdot\sin B$$

Use Sum & Difference Formulas

Find the exact value of sin(75⁰)

$$\sin\left(75^o\right) = \sin\left(30^o + 45^o\right)$$

$$\sin\left(75^o\right) = \sin 30^o \cos 45^o + \cos 30^o \sin 45^o$$

$$\sin\left(75^o\right) = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\sin\left(75^o\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\sin\left(75^o\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Use Sum & Difference Formulas

Find the exact value of: $\cos\left(\frac{7\pi}{12}\right)$

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$$

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Use Sum & Difference Formulas

Find the exact value of: $\tan\left(\frac{-13\pi}{12}\right)$

$$\tan\left(\frac{-13\pi}{12}\right) = -\tan\left(\frac{13\pi}{12}\right) \qquad -\tan\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)$$

$$= -\left[\frac{\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{3\pi}{4}\right)}{1-\tan\left(\frac{\pi}{3}\right)\cdot\tan\left(\frac{3\pi}{4}\right)}\right]$$

$$= -\left[\frac{\sqrt{3}+(-1)}{1-\sqrt{3}\cdot(-1)}\right]$$

Rationalize the denominator

$$= -\left[\frac{\sqrt{3}-1}{1+\sqrt{3}}\right]$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} \bullet \left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right) = \frac{1-2\sqrt{3}+3}{1-3} = \frac{4-2\sqrt{3}}{-2} = -2+\sqrt{3}$$

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Use Sum & Difference Formulas

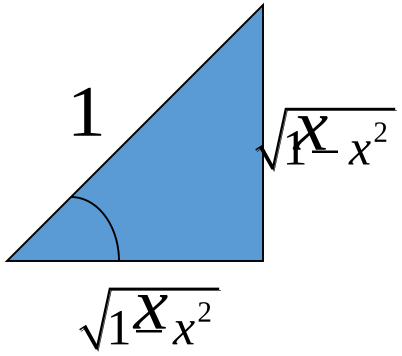
Simplfy: $\cos\left(\sin^{-1}x-\cos^{-1}x\right)$

$$=\cos\left(\sin^{-1}x\right)\cdot\cos\left(\cos^{-1}x\right)+\sin\left(\sin^{-1}x\right)\cdot\sin\left(\cos^{-1}x\right)$$

$$=\sqrt{1-x^2}\cdot x+x\cdot\sqrt{1-x^2}$$

$$=x\sqrt{1-x^2}+x\sqrt{1-x^2}$$

$$=2x\cdot\sqrt{1-x^2}$$



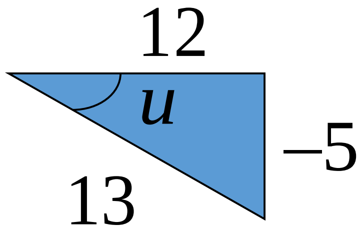
Use Sum & Difference Formulas

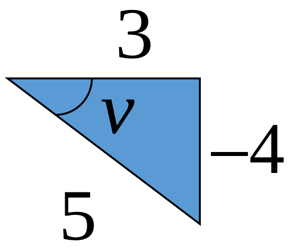
Find the exact value of sin(u+v) given the following:

$$\sin u = \frac{-5}{13} \;\; and \;\; \cos v = \frac{3}{5}, \; (u \; \& \; v \; are \; in \; Quad \; IV)$$

$$\sin(u+v) = \sin u \cdot \cos v + \cos u \cdot \sin v$$

$$\sin(u+v) = \frac{-5}{13} \cdot \frac{3}{5} + \cos u \cdot \sin v$$





$$\sin(u+v) = \frac{-5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{-4}{5}$$

$$\sin(u+v) = \frac{-15}{65} + \frac{-48}{65}$$

$$\sin(u+v) = -\frac{63}{65}$$

Verify Trig Identities

$$\sin(x+y)\bullet\sin(x-y) = \sin^2 x - \sin^2 y$$

$$[\sin x\cos y + \cos x\sin y]\cdot[\sin x\cos y - \cos x\sin y]$$

$$\sin^2 x\cos^2 y - \cos^2 x\sin^2 y$$

$$\sin^2 x\left(1-\sin^2 y\right) - \left(1-\sin^2 x\right)\sin^2 y$$

$$\sin^2 x - \sin^2 x\cdot\sin^2 y - \sin^2 y + \sin^2 x\cdot\sin^2 y$$

$$\sin^2 x - \sin^2 y = \sin^2 x - \sin^2 y$$

Solving Trig Equations on [0, 2π)

$$\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)=1$$

$$\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}+\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}=1$$

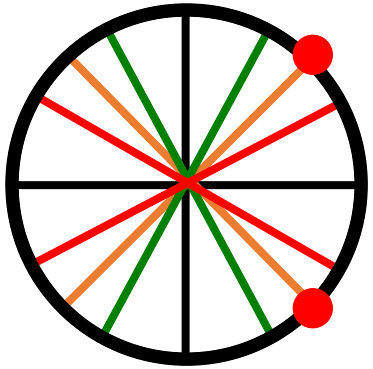
$$\cos x\cos\frac{\pi}{4}+\cos x\cos\frac{\pi}{4}=1$$

$$2\cos x\cos\frac{\pi}{4}=1$$

$$2\bullet\cos x\bullet\left(\frac{\sqrt{2}}{2}\right)=1$$

$$\sqrt{2}\cos x=1$$

$$\cos x=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$



$$x=\frac{\pi}{4}\ or\ \ x=\frac{7\pi}{4}$$

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Double-Angle Formulas

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Double-Angle Formulas

$$\sin(2x) = \sin(x+x)$$
$$\sin(2x) = \sin x\cos x + \cos x\sin x$$
$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos(x+x)$$
$$\cos(2x) = \cos x\cos x - \sin x\sin x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$
$$\cos(2x) = 1 - \sin^2 x - \sin^2 x$$
$$\cos(2x) = 1 - 2\sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$
$$\cos(2x) = \cos^2 x - (1 - \cos^2 x)$$
$$\cos(2x) = \cos^2 x - 1 + \cos^2 x$$
$$\cos(2x) = 2\cos^2 x - 1$$

Double-Angle Formulas

$$\tan(2x) = \tan(x+x)$$

$$\tan(2x) = \frac{\tan x + \tan x}{1 - \tan x \cdot \tan x}$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

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Re-cap

Double-Angle Formulas

$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$$

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Half-Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

Note: the signs of $\sin\left(\frac{u}{2}\right)$ *and* $\cos\left(\frac{u}{2}\right)$ *depend on the quadrant in which* $\frac{u}{2}$ *lies.*

Solve for x

$$\sin 2x + \cos x = 0$$

$$2\sin x \cos x + \cos x = 0$$

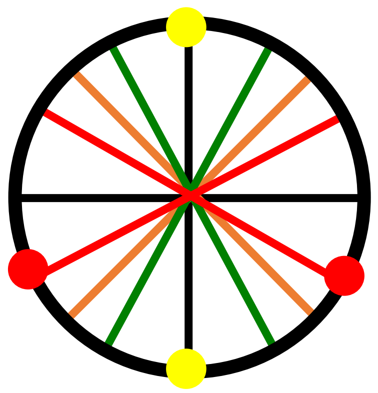
$$\cos x(2\sin x + 1) = 0$$

$$x = \frac{\pi}{2} + n\pi$$
$$x = \frac{7\pi}{6} + 2n\pi$$
$$x = \frac{11\pi}{6} + 2n\pi$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{2} + n\pi$$



$$2\sin x + 1 = 0$$

$$2\sin x = -1$$

$$\sin x = \frac{-1}{2}$$

$$x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{11\pi}{6} + 2n\pi$$

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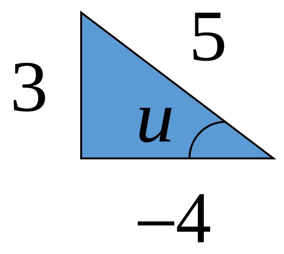
Use Double-Angle Formulas

Find the exact value of sin(2u), cos(2u) & tan(2u)

$$given\ \sin u = \frac{3}{5},\ \frac{\pi}{2} < u < \pi$$



$$\sin(2u) = 2\sin u \cdot \cos u$$

$$\sin(2u) = 2 \cdot \frac{3}{5} \cdot \frac{-4}{5} = \frac{-24}{25}$$

$$\cos(2u) = 1 - 2\sin^2 u$$

$$\cos(2u) = 1 - 2\left(\frac{3}{5}\right)^2$$

$$\cos(2u) = 1 - 2 \cdot \frac{9}{25}$$

$$\cos(2u) = 1 - \frac{18}{25} = \frac{7}{25}$$

$$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan(2x) = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$$

$$\tan(2x) = \frac{-\frac{3}{2}}{1-\frac{9}{16}}$$

$$\tan(2x) = \frac{-\frac{3}{2}}{\frac{7}{16}}$$

$$\tan(2x) = -\frac{3}{2} \cdot \frac{16}{7} = \frac{-24}{7}$$

Use Half-Angle Formulas

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Find the exact value of: $\sin\left(\frac{5\pi}{12}\right)$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\frac{5\pi}{6}}{2}\right)$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$= \sqrt{\frac{1-\cos\left(\frac{5\pi}{6}\right)}{2}}$$

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

Use Half-Angle Formulas

Find the exact value of: $\tan\left(\frac{7\pi}{8}\right)$

$$\tan\left(\frac{7\pi}{8}\right) = \tan\left(\frac{\frac{7\pi}{4}}{2}\right)$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

$$= \frac{1-\cos\left(\frac{7\pi}{4}\right)}{\sin\left(\frac{7\pi}{4}\right)}$$

$$= \frac{1-\left(\frac{\sqrt{2}}{2}\right)}{-\frac{\sqrt{2}}{2}}$$

$$= \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} \bullet \frac{-2}{-2}$$

$$= \frac{-2+\sqrt{2}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2}+2}{2}$$

$$= -\sqrt{2}+1 = 1-\sqrt{2}$$

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Solve for x

$$\sin\left(\frac{x}{2}\right) + \cos x - 1 = 0$$

$$\pm\sqrt{\frac{1-\cos x}{2}} + \cos x - 1 = 0$$

$$\pm\sqrt{\frac{1-\cos x}{2}} = 1 - \cos x$$

$$\left[\pm\sqrt{\frac{1-\cos x}{2}}\right]^2 = \left[1 - \cos x\right]^2$$

$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$

$$1-\cos x = 2 - 4\cos x + 2\cos^2 x$$

$$0 = 1 - 3\cos x + 2\cos^2 x$$

$$0 = \left(1 - 2\cos x\right)\left(1 - \cos x\right)$$

$$0 = 1 - 2\cos x$$

$$-1 = -2\cos x$$

$$-1 = -2\cos x$$

$$\frac{1}{2} = \cos x$$

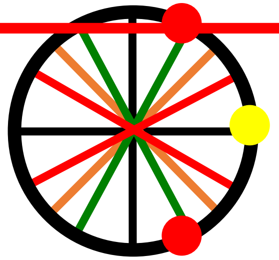
$$\frac{\pi}{3} + 2n\pi = x$$

$$\frac{5\pi}{3} + 2n\pi = x$$

$$0 = 1 - \cos x$$

$$\cos x = 1$$

$$x = 2n\pi$$



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Product-to-Sum Formulas

$$\sin u \cdot \sin v = \tfrac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\cos u \cdot \cos v = \tfrac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\sin u \cdot \cos v = \tfrac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

$$\cos u \cdot \sin v = \tfrac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

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Use Power-Reducing Formulas

Rewrite $\sin^4 x$ in terms of the first power of cosine.

$$\sin^4(x) = \left(\sin^2 x\right)^2$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \left(\tfrac{1}{4}\right)\left(1-\cos 2x\right)^2$$

$$= \left(\tfrac{1}{4}\right)\left(1-2\cos 2x+\cos^2 2x\right)$$

$$= \left(\tfrac{1}{4}\right)\left(1-2\cos 2x+\tfrac{1+\cos 4x}{2}\right)$$

$$= \left(\tfrac{1}{8}\right)\left(2-4\cos 2x+1+\cos 4x\right)$$

$$= \left(\tfrac{1}{8}\right)\left(3-4\cos 2x+\cos 4x\right)$$