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Cramer’s Rule

Cramer's Rule - 2 x 2

- Cramer's Rule relies on determinants.
- Consider the system below with variables *x* and *y*:

$$a_1x + b_1y = C_1$$
$$a_2x + b_2y = C_2$$

Cramer's Rule - 2 x 2

- The formulae for the values of *x* and *y* are shown below. The numbers inside the determinants are the coefficients and constants from the equations.

$$x = \frac{\begin{vmatrix} C_1 & b_1 \\ C_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & C_1 \\ a_2 & C_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

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Cramer's Rule

- Example:

  Solve the system:

$$3x - 2y = 10$$
$$4x + y = 6$$

$$x = \frac{\begin{vmatrix} 10 & -2 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix}} = \frac{22}{11} = 2$$

$$y = \frac{\begin{vmatrix} 3 & 10 \\ 4 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix}} = \frac{-22}{11} = -2$$

The solution is

(2, -2)

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Cramer's Rule - 3 x 3

- Consider the 3-equation system below with variables *x*, *y* and *z*:

$$a_1x + b_1y + c_1z = C_1$$
$$a_2x + b_2y + c_2z = C_2$$
$$a_3x + b_3y + c_3z = C_3$$

Cramer's Rule - 3 x 3

- The formulae for the values of *x*, *y* and *z* are shown below. Notice that all three have the same denominator.

$$x = \frac{\begin{vmatrix} C_1 & b_1 & c_1 \\ C_2 & b_2 & c_2 \\ C_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & C_1 & c_1 \\ a_2 & C_2 & c_2 \\ a_3 & C_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & C_1 \\ a_2 & b_2 & C_2 \\ a_3 & b_3 & C_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

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Cramer's Rule

- Not all systems have a definite solution. If the determinant of the coefficient matrix is zero, a solution cannot be found using Cramer's Rule

  due to division by zero.
- When the solution cannot be determined, one of two conditions exists:
  - The planes graphed by each equation are parallel and there are no solutions.
  - The three planes share one line (like three pages of a book share the same spine) or represent the same plane, in which case there are infinite solutions.

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Cramer's Rule

- Example:
  Solve the system

$$\begin{aligned} 3x - 2y + z &= 9 \\ x + 2y - 2z &= -5 \\ x + y - 4z &= -2 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 9 & -2 & 1 \\ -5 & 2 & -2 \\ -2 & 1 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{-23}{-23} = 1$$

$$y = \frac{\begin{vmatrix} 3 & 9 & 1 \\ 1 & -5 & -2 \\ 1 & -2 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{69}{-23} = -3$$

Cramer's Rule

- Example, continued:

$$3x - 2y + z = 9$$
$$x + 2y - 2z = -5$$
$$x + y - 4z = -2$$

$$z = \frac{\begin{vmatrix} 3 & -2 & 9 \\ 1 & 2 & -5 \\ 1 & 1 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{0}{-23} = 0$$

The solution is

(1, -3, 0)