

Inverse Matrices (2 x 2)

How to find the inverse of a 2x2 matrix



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Inverse of a number

When we are talking about our natural numbers, the inverse of a number is its reciprocal. When we multiply a number by its inverse we get 1.

For example:



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Inverse of a matrix

What do you think we would get if we multiplied a matrix by its inverse? Try it on your calculator.

$$A \times A^{-1} = I$$

A matrix multiplied by its inverse always gives us an identity matrix.



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Inverse of a matrix

Not all matrices have an inverse.

If the determinant of a matrix is 0,
then it has no inverse and is said to be
SINGULAR.

All others are said to be **NON-SINGULAR**



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Inverse of a matrix

Which of these have an inverse?



$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$





Finding Inverses 2x2

$$A \times A^{-1} = I$$

$$A = \begin{bmatrix} 8 & -10 \\ -3 & 4 \end{bmatrix}$$

$$\text{Let } A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{So } AA^{-1} = I \quad \Rightarrow \begin{bmatrix} 8 & -10 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying out gives..

$$\Rightarrow \begin{bmatrix} 8a - 10c & 8b - 10d \\ -3a + 4c & -3b + 4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$8a - 10c = 1$$

$$8b - 10d = 0$$

$$-3a + 4c = 0$$

$$-3b + 4d = 1$$

Can you solve these to work out A^{-1} ?

$$A^{-1} = \begin{bmatrix} 2 & 5 \\ 1.5 & 4 \end{bmatrix}$$



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Finding Inverses 2x2

There is an alternative method.

ad-bc represents $\det(A)$. What would happen if this was zero?

In words:

- Take the original matrix.
- Switch a and d .
- Change the signs of b and c .
- Multiply the new matrix by $\frac{1}{\det(A)}$ over the determinant of the original matrix.

$$\frac{1}{\det(A)}$$

Finding Inverses 2x2

Example: Find the inverse of A.

$$A = \begin{bmatrix} 2 & 4 \\ -4 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(2)(-10) - (-4)(4)} \begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$$



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Finding Inverses 2x2

Find the inverses of A , B and C , where

$$A = \begin{pmatrix} 7 & 19 \\ 2 & 6 \end{pmatrix},$$

$$B = \begin{pmatrix} 5 & \frac{1}{2} \\ 20 & 2 \end{pmatrix},$$

$$C = \begin{pmatrix} a & -4 \\ 1 & 3 \end{pmatrix}.$$