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Matrices and Systems of Linear Equations

Matrices and System of Linear Equations

A m x n matrix is read “m by n” and has m rows (horizontal lines) and n columns (vertical lines)

What order does this matrix have?

$$\begin{vmatrix} 5 & 0 \\ 2 & -2 \\ -7 & 4 \end{vmatrix}$$

3 x 2

3 rows by 2 columns

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Use Gaussian Elimination with back-substitution to solve.

$$
\begin{aligned}
x - 2y + 3z &= 9 \\
-x + 3y &= -4 \\
2x - 5y + 5z &= 17
\end{aligned}
$$

First, we want to rewrite these equations in matrix form.

$$
\left|\begin{matrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{matrix}\right|
\qquad
\left|\begin{matrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{matrix}\right|
$$

Where

x = a

y = b

z = c

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We are going to do 2 problems in this one example. First, we are going to do Gaussian elimination with back-substitution. Then we are going to do Gauss-Jordon elimination.

$$\begin{vmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{vmatrix}$$

Step 1: make sure our matrix begins with a 1 in a_{11}.

It does. If it had not, we would either trade it with a line that does or divide each term in the row by the coefficient in front of x.

Now get a_{21} to be 0 by adding $R_1 + R_2$

-2 4 -6 -18

$$M_{-2}\begin{vmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{vmatrix}$$

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Now get $a_{31} = 0$ by $-2R_1 + R_3$

$$\begin{vmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{vmatrix}$$

Now we want a_{22} to be a 1. It already is so now make $a_{32} = 0$

We can do this by adding $R_2 + R_3$.

$$\begin{vmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

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$$\left|\begin{array}{cccc} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right| \Rightarrow \begin{array}{r} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{array}$$

Now that we found z to be 2, we can back-substitute to find y and z.

y + 3(2) = 5
y = -1

x - 2(-1) + 3(2) = 9
x = 1

The final answer is (1, -1, 2)

Once again, this is called Gausian-elimination with back-substitution.

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Gauss-Jordon Elimination continues this last process

$$\left|\begin{matrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{matrix}\right|$$

$$\left|\begin{matrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{matrix}\right|$$

Next, we need to make $a_{23} = 0$

How are we going to do this?

Multiply row 3 by -3 and add it to row 2. **$R_2 + -3R_3$**

This gives us

$$\begin{vmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

Now let's make $a_{12} = 0$ by multiplication row 2 by 2 and adding it to row 1.
or $2R_2 + R_1$

$$\begin{vmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

Now make $a_{13} = 0$
How do we do this?

$-3R_3 + R_1$

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This gives us a final answer of:

$$\left|\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right| \Rightarrow$$

x = 1

y = -1

z = 2