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# **Matrices and Systems of Linear Equations**



## Matrices and System of Linear Equations

**A  $m \times n$  matrix is read “m by n” and has m rows (horizontal lines) and n columns (vertical lines)**

**What order does this matrix have?**

$$\begin{vmatrix} 5 & 0 \\ 2 & -2 \\ -7 & 4 \end{vmatrix}$$

**3 x 2**

**3 rows by 2 columns**



**Use Gaussian Elimination with back-substitution to solve.**

$$\begin{aligned}x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17\end{aligned}$$

First, we want to rewrite these equations in matrix form.

$$\left| \begin{array}{cccc} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array} \right| \quad \left| \begin{array}{cccc} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right| \quad \begin{array}{l} \text{Where} \\ \mathbf{x = a} \\ \mathbf{y = b} \\ \mathbf{z = c} \end{array}$$



We are going to do 2 problems in this one example. First, we are going to do Gaussian elimination with back-substitution. Then we are going to do Gauss-Jordan elimination.

$$\left[ \begin{array}{cccc} \textcircled{1} & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array} \right]$$

Step 1: make sure our matrix begins with a 1 in  $a_{11}$ .

It does. If it had not, we would either trade it with a line that does or divide each term in the row by the coefficient in front of x.

Now get  $a_{21}$  to be 0 by adding  $R_1 + R_2$

$$M_{-2} \left[ \begin{array}{cccc} -2 & 4 & -6 & -18 \\ 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array} \right]$$



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Now get  $a_{31} = 0$   
by  $-2R_1 + R_3$

$$\left\langle \begin{array}{cccc} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array} \right|$$

Now we want  $a_{22}$  to be a 1. It already is so now make  $a_{32} = 0$

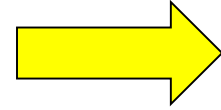
We can do this by adding  $R_2 + R_3$ .

$$\left| \begin{array}{cccc} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array} \right| = \left| \begin{array}{cccc} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array} \right|$$



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$$\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}$$



$$\mathbf{x} - 2\mathbf{y} + 3\mathbf{z} = 9$$

$$\mathbf{y} + 3\mathbf{z} = 5$$

$$\mathbf{z} = 2$$

**Now that we found  $z$  to be 2, we can back-substitute to find  $y$  and  $x$ .**

$$\begin{aligned} \mathbf{y} + 3(2) &= 5 \\ \mathbf{y} &= -1 \end{aligned}$$

$$\begin{aligned} \mathbf{x} - 2(-1) + 3(2) &= 9 \\ \mathbf{x} &= 1 \end{aligned}$$

**The final answer is  $(1, -1, 2)$**

**Once again, this is called Gaussian-elimination with back-substitution.**



$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right|$$

$$\left| \begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array} \right|$$

**Next, we need to make  $a_{23} = 0$**

**How are we going to do this?**

**Multiply row 3 by -3 and add  
it to row 2.  $R_2 + -3R_3$**

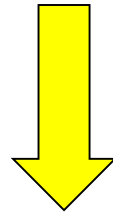


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This gives us

$$\left| \begin{array}{cccc} 1 & -2 & 3 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right|$$



$$\left| \begin{array}{cccc} 1 & 0 & 3 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right|$$

Now let's make  $a_{12} = 0$   
by multiplication row 2 by 2 and  
adding it to row 1.

or  $2R_2 + R_1$

Now make  $a_{13} = 0$   
How do we do this?

$-3R_3 + R_1$





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**This gives us a final answer of:**

$$\left| \begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right| \longrightarrow \begin{array}{l} \mathbf{x} = 1 \\ \mathbf{y} = -1 \\ \mathbf{z} = 2 \end{array}$$