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MATRICES

MATRIX OPERATIONS



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About Matrices

- **A matrix is a rectangular arrangement of numbers in rows and columns. Rows run horizontally and columns run vertically.**
- **The dimensions of a matrix are stated “ $m \times n$ ” where ‘ m ’ is the number of rows and ‘ n ’ is the number of columns.**



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Square Matrices

- **Two matrices are considered Square if they have the same number of rows and columns (the same dimensions) **AND** all their corresponding elements are exactly the same.**



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Special Matrices

Some matrices have special names because of what they look like.

- a) Row matrix: only has 1 row.
- b) Column matrix: only has 1 column.
- c) Square matrix: has the same number of rows and columns.
- d) Zero matrix: contains all zeros.



Matrix Addition

- You can add or subtract matrices **if** they have the same dimensions (same number of rows and columns).
- To do this, you add (or subtract) the corresponding numbers (numbers in the same positions).

Matrix Addition

Example:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 7 & -1 \\ -2 & 0 \end{bmatrix}$$

Scalar Multiplication

- **To do this, multiply each entry in the matrix by the number outside (called the scalar). This is like distributing a number to a polynomial.**

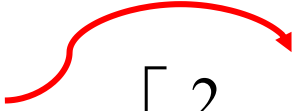
Scalar Multiplication

Example:

$$4 \begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ 20 & 0 \\ 4 & -12 \end{bmatrix}$$

Matrix Multiplication

- **Matrix Multiplication is NOT Commutative!**
Order matters!
- You can multiply matrices only if the number of **columns** in the first matrix equals the number of **rows** in the second matrix.

2 columns 

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} \leftarrow \text{2 rows}$$

Matrix Multiplication

- **Take the numbers in the first row of matrix #1. Multiply each number by its corresponding number in the first column of matrix #2. Total these products.**

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

$$2 \times 1 + 3 \times 3 = 11$$


The result, 11, goes in row 1, column 1 of the answer. Repeat with row 1, column 2; row 1 column 3; row 2, column 1; ...

Matrix Multiplication

- Notice the dimensions of the matrices and their product.

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} = \begin{bmatrix} 11 & 8 & -15 \\ 13 & 34 & -30 \\ -12 & -46 & 35 \end{bmatrix}$$

3 x 2 2 x 3 3 x 3



Matrix Multiplication

- Another example:

$$\begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 10 & -5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ -45 \\ 60 \end{bmatrix}$$

3×2 2×1 3×1