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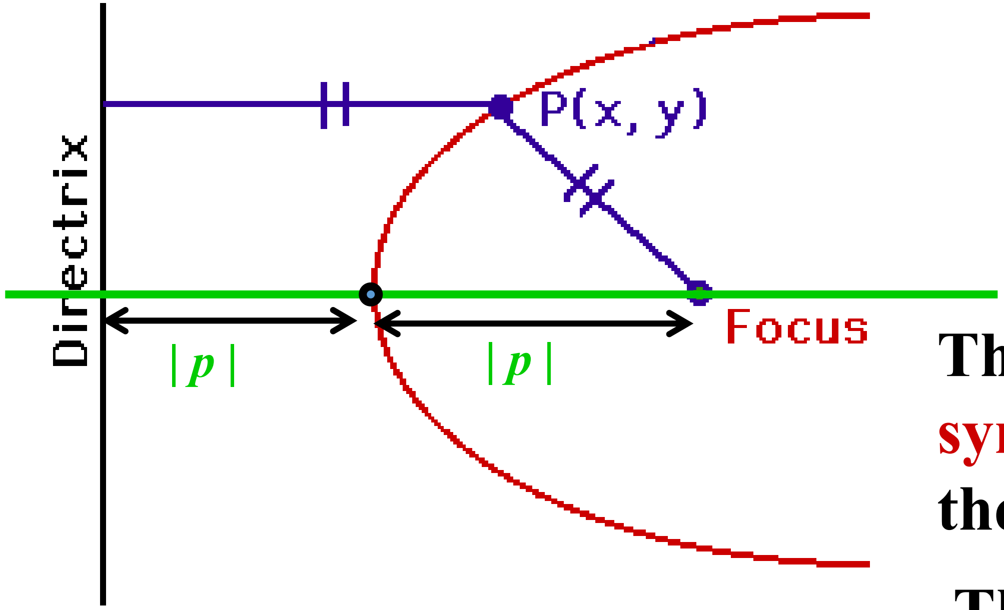
Parabolas

The Parabola

The parabola is the locus of all points in a plane that are the same distance from a line in the plane, the **directrix**, as from a fixed point in the plane, the **focus**.



Point Focus = Point Directrix
PF = PD

The parabola has one **axis of symmetry**, which intersects the parabola at its **vertex**.

The distance from the vertex to the focus is $|p|$.

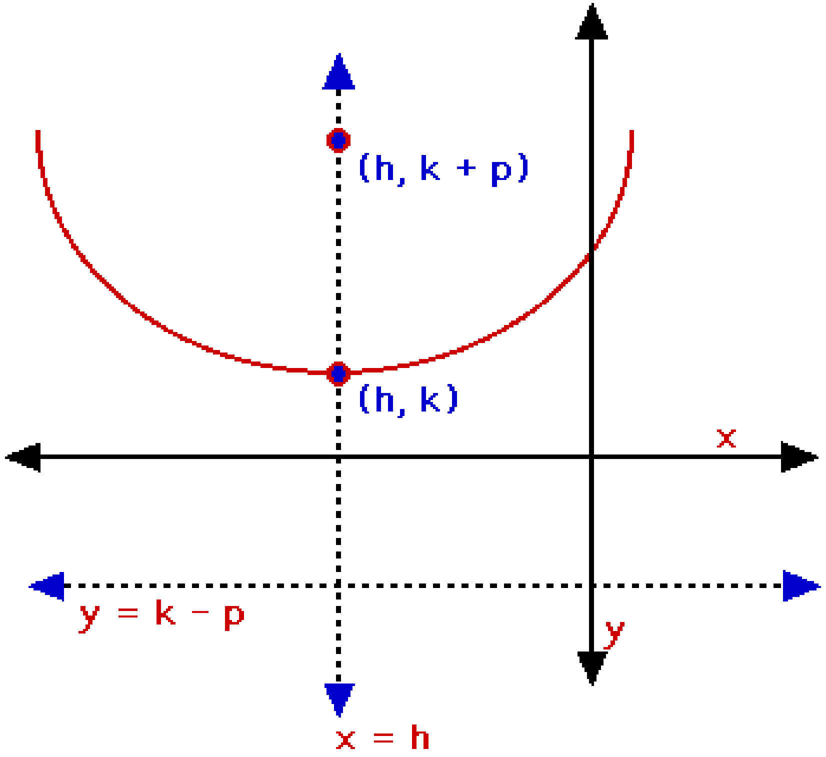
The distance from the directrix to the vertex is also $|p|$.

The Standard Form of the Equation with Vertex (*h*, *k*)

For a parabola with the axis of symmetry **parallel to the *y*-axis** and **vertex at (*h*, *k*)**, the standard form is …

$$(x - h)^2 = 4p(y - k)$$

- The equation of the axis of symmetry is $x = h$.
- The coordinates of the focus are $(h, k + p)$.
- The equation of the directrix is $y = k - p$.
- When p is positive, the parabola opens upward.
- When p is negative, the parabola opens downward.



The Standard Form of the Equation with Vertex (h, k)

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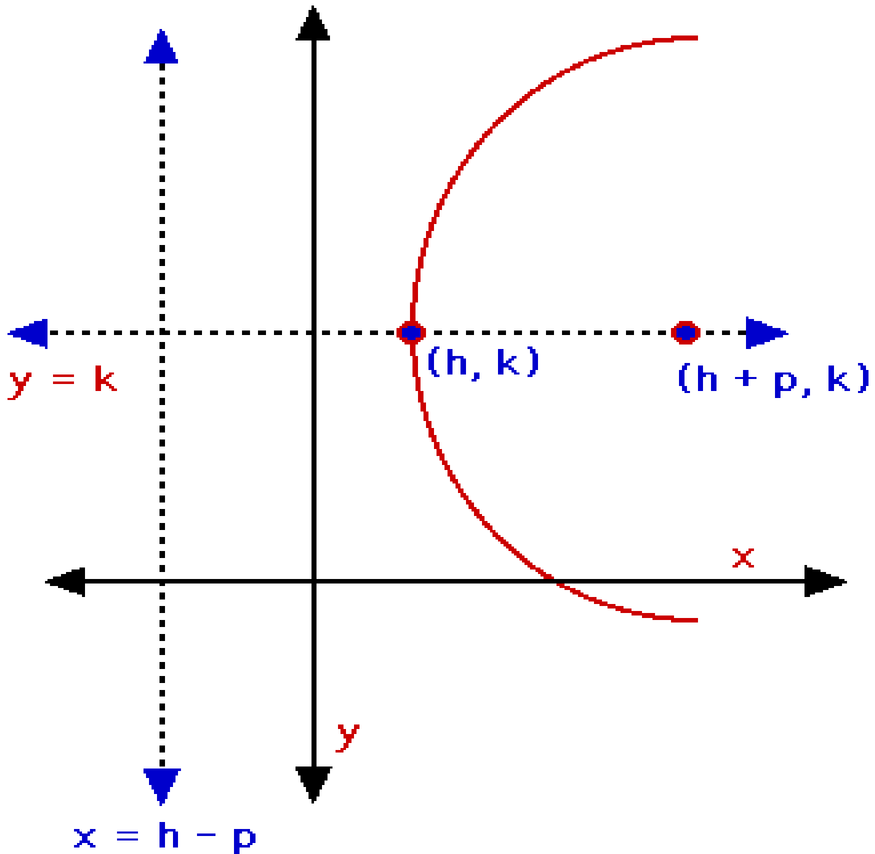
For a parabola with an axis of symmetry parallel to the x-axis and a vertex at (h, k), the standard form is:

$$(y - k)^2 = 4p(x - h)$$

- The equation of the axis of symmetry is $y = k$.
- The coordinates of the focus are $(h + p, k)$.
- The equation of the directrix is $x = h - p$.
- When p is positive, the parabola opens to the right.
- When p is negative, the parabola opens to the left.



Finding the Equations of Parabolas

Write the equation of the parabola with a focus at (3, 5) and the directrix at $x = 9$, in standard form and general form

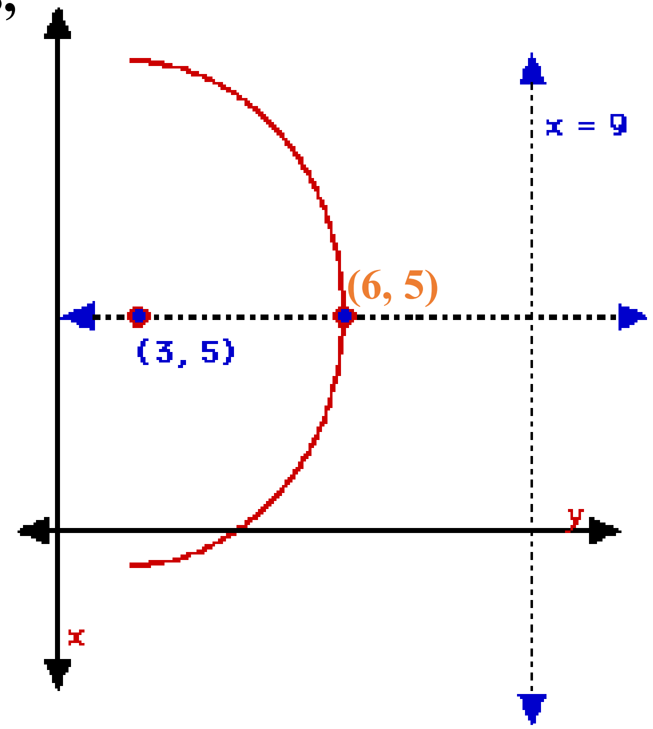
The distance from the focus to the directrix is 6 units, therefore, $2p = -6$, $p = -3$. Thus, the vertex is (6, 5).

The axis of symmetry is parallel to the *x*-axis:

$$(y - k)^2 = 4p(x - h) \qquad h = 6 \text{ and } k = 5$$

$$(y - 5)^2 = 4(-3)(x - 6)$$

$$(y - 5)^2 = -12(x - 6) \qquad \text{Standard form}$$



Finding the Equations of Parabolas

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Find the equation of the parabola that has a minimum at (-2, 6) and passes through the point (2, 8).

The axis of symmetry is parallel to the y-axis.

The vertex is (-2, 6), therefore, $h = -2$ and $k = 6$.

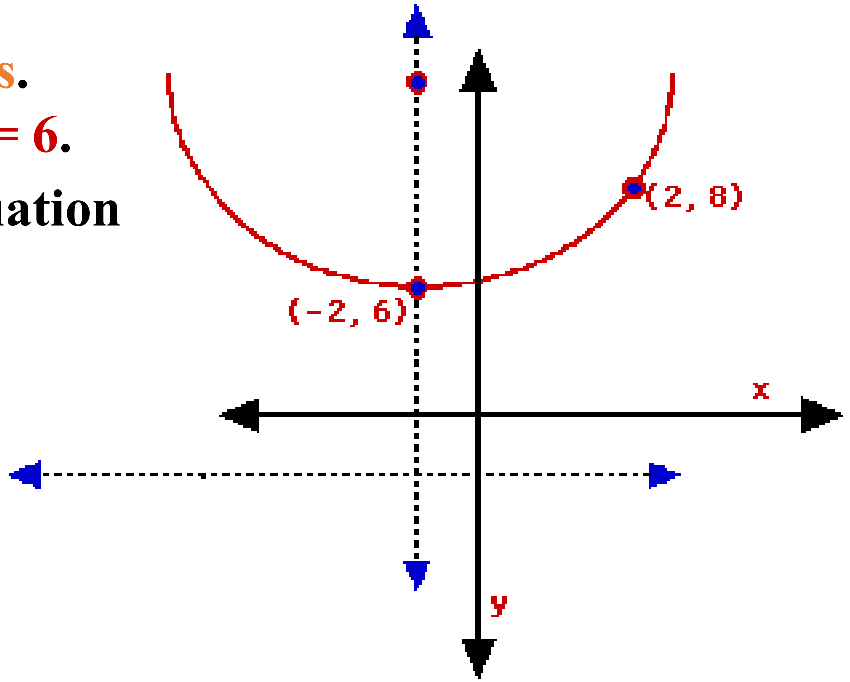
Substitute into the standard form of the equation and solve for p:

$$(x - h)^2 = 4p(y - k) \qquad x = 2 \text{ and } y = 8$$

$$(2 - (-2))^2 = 4p(8 - 6)$$

$$16 = 8p$$

$$2 = p$$



$$(x - h)^2 = 4p(y - k)$$

$$(x - (-2))^2 = 4(2)(y - 6)$$

$$(x + 2)^2 = 8(y - 6) \qquad \text{Standard form}$$

Analyzing a Parabola

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Find the coordinates of the vertex and focus, the equation of the directrix, the axis of symmetry, and the direction of opening of $2x^2 + 4x - 2y + 6 = 0$.

$2x^2 + 4x - 2y + 6 = 0$

$2(x^2 + 2x + \underline{\ 1\ }) = 2y - 6 + \underline{2(1)}$

$2(x + 1)^2 = 2(y - 2)$

$(x + 1)^2 = (y - 2)$

$4p = 1$

$p = \frac{1}{4}$

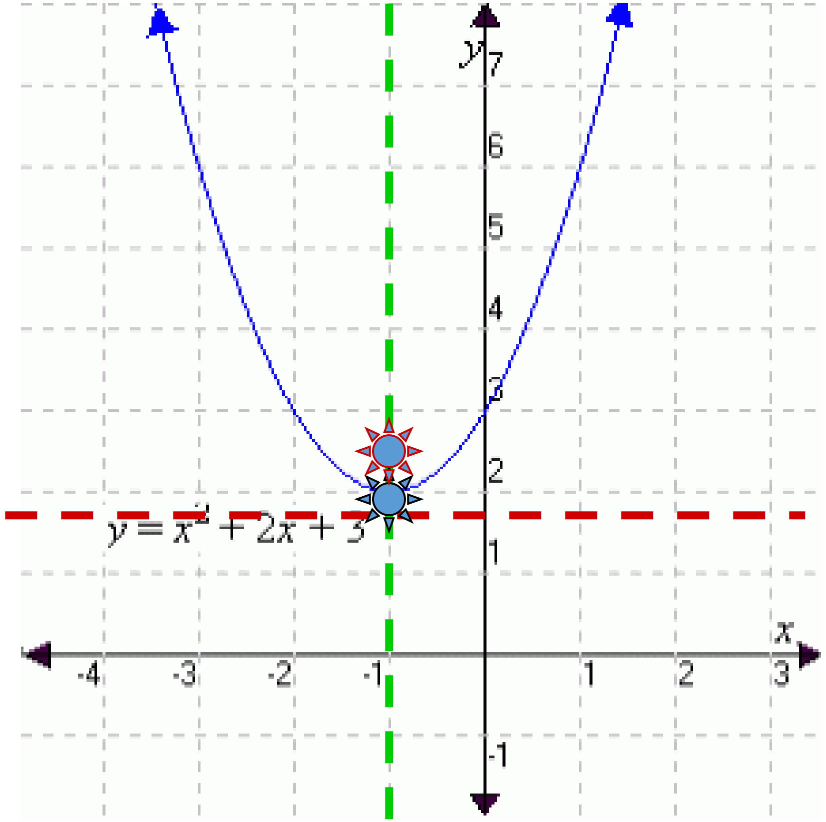
The parabola opens to upward.

The vertex is (-1, 2).

The focus is (-1, 2 ¼).

The Equation of directrix is $y = 1\frac{3}{4}$.

The axis of symmetry is x = -1 .



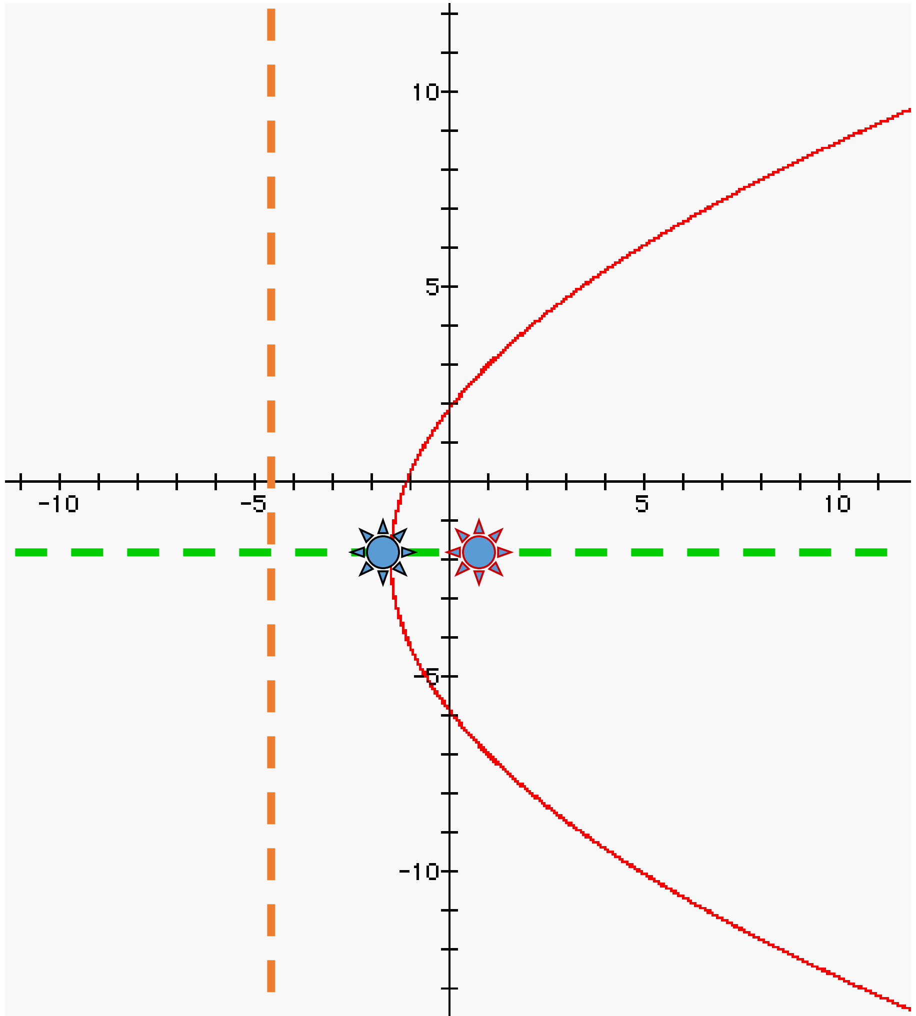
Graphing a Parabola

$y^2 - 10x + 4y - 16 = 0$

$y^2 + 4y + \underline{\ 4\ } = 10x + 16 + \underline{\ 4\ }$

$(y + 2)^2 = 10x + 20$

$(y + 2)^2 = 10(x + 2)$



Horizontally oriented (right)

Vertex @ (-2, -2)

Line of Symmetry y = -2

P = 2.5

focus @ (0.5, -3)

Directrix X = - 4.5